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Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (14 points) Use the Chain Rule to find the partial derivatives  $\frac{\partial w}{\partial u}$  and  $\frac{\partial w}{\partial v}$ , where

$$w = 3xy + y^3 - 1, \quad x = 2u + v^2, \quad y = u^3 - 3v,$$

when  $u = 1$  and  $v = 2$ .

2. (14 points) Find the velocity and position of a particle that has the given acceleration and the given initial velocity and position:

$$\mathbf{a}(t) = 2t \mathbf{i} - 10 \mathbf{k}, \quad \mathbf{v}(0) = 3 \mathbf{j}, \quad \mathbf{r}(0) = \mathbf{0}.$$

3. (15 points) Let  $f(x, y) = -\sqrt{9 - x^2 - 4y^2}$ .

- Find the domain and range of  $f(x, y)$
- Sketch a few level curves of  $f(x, y)$ .
- Sketch the surface  $z = f(x, y)$ .

4. (15 points) Find an equation of the tangent plane to the graph of the function

$$f(x, y) = (x + 2y) \cos(3xy)$$

at the point  $(0, 1, 2)$ .

5. (15 points) Let  $g(x, y, z) = e^z - xy^2 + z \sin(\pi x)$ .

- Find the directional derivative of  $g$  in the direction of the vector  $\mathbf{v} = \langle 1, 1, 1 \rangle$  at the point  $P(1, -2, 0)$ .
- In which direction does  $g$  have the maximum rate of change at that point? What is its maximum rate of change there?

6. (15 points) Find all the local minima, maxima, and saddle point points of the function

$$f(x, y) = 5x^2 - 4xy + y^2 - 4x + 6y + 10.$$

7. (15 points) The total resistance  $R$  of two resistors with resistances  $R_1$  and  $R_2$  connected in parallel is given by the following well-known formula:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose that  $R_1$  and  $R_2$  are measured to be 300 and 600 ohms, respectively, with an error of  $\pm 15$  ohms in each measurement. Use differentials to estimate the maximum error in ohms in the calculated value of  $R$ .