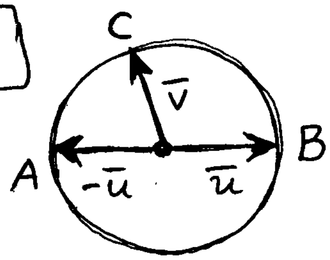
$\boxed{12.3.17}$ Yes, it is a coincidence that $\vec{V}_1+\vec{V}_2 \perp \vec{V}_1-\vec{V}_2$ since it is not always true. Use dot product to see when it is true:

$$(\vec{V}_1+\vec{V}_2)\cdot(\vec{V}_1-\vec{V}_2) = \vec{V}_1\cdot\vec{V}_1 + \underbrace{\vec{V}_2\cdot\vec{V}_1 - \vec{V}_1\cdot\vec{V}_2}_{=0} - \vec{V}_2\cdot\vec{V}_2 .$$

$$= |\vec{V}_1|^2 - |\vec{V}_2|^2 = 0 \text{ only if } |\vec{V}_1| = |\vec{V}_2|$$

(Not always true)

$\boxed{12.3.18}$



Show that $\vec{CA} \perp \vec{CB}$:

By triangle rule: $\vec{CA} = -\vec{u} - \vec{V}$

$\vec{CB} = \vec{u} - \vec{V}$

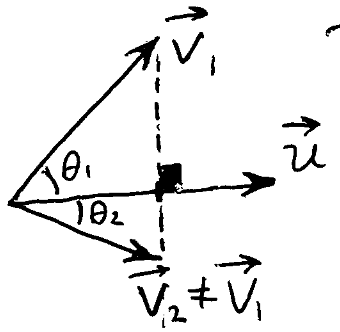
$$\vec{CA}\cdot\vec{CB} = (-\vec{u}-\vec{V})\cdot(\vec{u}-\vec{V}) = -|u|^2 + \vec{u}\cdot\vec{V} - \vec{V}\cdot\vec{u} + |V|^2$$

$$= |V|^2 - |u|^2 = 0 \quad \text{because } |u| = |V| = R \text{ radius}$$

■

$\boxed{12.3.28}$ $\vec{u}\cdot\vec{V}_1 = \vec{u}\cdot\vec{V}_2$ does not mean that $\vec{V}_1 = \vec{V}_2$

It means that projections of $\vec{V}_1$ & $\vec{V}_2$ onto $\vec{u}$ are the same, which does not require $\vec{V}_1 = \vec{V}_2$:



Note that $\vec{V}_1\cdot\vec{u} = \vec{V}_2\cdot\vec{u}$, but $\vec{V}_1 \neq \vec{V}_2$ since $|\vec{V}_1|\cos\theta_1 = |\vec{V}_2|\cos\theta_2$ does not even require $|\vec{V}_1| = |\vec{V}_2|$

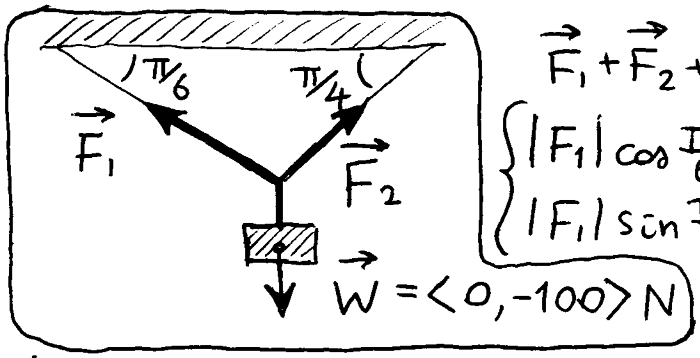
$\boxed{12.4.48}$ Volume $= |\vec{u}\cdot\vec{V}\times\vec{W}|$ where $\vec{u} = AB = \langle 1,2,0\rangle$

$\vec{V} = AC = \langle 0,-3,2\rangle$

$\vec{W} = AD = \langle 3,-4,5\rangle$

$$|\vec{u}\cdot\vec{V}\times\vec{W}| = \begin{vmatrix} u_1 & u_2 & u_3 \\ V_1 & V_2 & V_3 \\ W_1 & W_2 & W_3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 0 \\ 0 & -3 & 2 \\ 3 & -4 & 5 \end{vmatrix} = 1\cdot(-15+8) + 2\cdot 6 + 0 = 12 - 7 = \boxed{5}$$

12.2.45



$$\vec{F}_1 + \vec{F}_2 + \vec{W} = 0$$

$$\begin{cases} |F_1| \cos\frac{\pi}{6} = |F_2| \cos\frac{\pi}{4} \\ |F_1| \sin\frac{\pi}{6} + |F_2| \sin\frac{\pi}{4} = 100 \end{cases}$$

$$\begin{cases} |F_1|\frac{\sqrt{3}}{2} = |F_2|\frac{1}{\sqrt{2}} \mapsto |F_2| = \sqrt{3/2}\,|F_1| \\ |F_1|\frac{1}{2} + |F_2|\frac{1}{\sqrt{2}} = 100 \to |F_1|\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 100 \to \end{cases}$$

$$\boxed{|F_1| = \frac{200}{1+\sqrt{3}} \qquad |F_2| = \sqrt{\frac{3}{2}}\,\frac{200}{1+\sqrt{3}}}$$

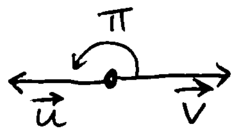
$$\vec{F}_1 = \langle -|F_1|\cos\frac{\pi}{6}, |F_1|\sin\frac{\pi}{6} \rangle$$

$$= \langle -100\frac{\sqrt{3}}{1+\sqrt{3}}, \frac{100}{1+\sqrt{3}} \rangle = \boxed{\langle -63.4, 36.6 \rangle}$$

$$\vec{F}_2 = \langle |F_2|\cos\frac{\pi}{4}, |F_2|\sin\frac{\pi}{4} \rangle = \langle 100\frac{\sqrt{3}}{1+\sqrt{3}}, 100\frac{\sqrt{3}}{1+\sqrt{3}} \rangle = \boxed{\langle 63.4, 63.4 \rangle}$$

12.3.1 $\vec{v} = \langle 2, -4, \sqrt{5} \rangle$, $\vec{u} = \langle -2, 4, -\sqrt{5} \rangle$

- $\vec{v} \cdot \vec{u} = 2(-2) - 4 \cdot 4 - \sqrt{5}\sqrt{5} = -4 - 16 - 5 = -25$
- $|\vec{v}| = \sqrt{4+16+5} = 5$
- $|\vec{u}| = \sqrt{4+16+5} = 5$
- $\cos\theta = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|} = -\frac{25}{5 \cdot 5} = -1 \mapsto \theta = \pi$



- $\text{proj}_v \vec{u} = \vec{u} \cdot \hat{v}\,\hat{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\vec{v} = \frac{-25}{25}\vec{v} = \boxed{-\vec{v} = \vec{u}}$

Note: $\vec{u} = -\vec{v}$, so $\vec{u}$ projected onto $\vec{v}$ gives $-\vec{v} = \vec{u}$