Read each problem carefully. Show all work for each problem. No calculators or notes.
You must remain in the room until you have completed the test.
All cell phones including smart-phones should be turned off.

1. **(14pts)** Use vector algebra to show that a parallelogram with diagonals of equal lengths is a rectangle.

2. **(14pts)** Consider two objects with trajectories given below, as vector functions of real time "t". Answer the following questions: (a) Do the two trajectories intersect? (b) Do these objects collide?

$\vec{r}_1(t) = \langle t-1,\ 2t+2,\ 3t+2 \rangle$ and $\vec{r}_2(t) = \langle 2t-1,\ 2t,\ 1+5t \rangle$

3. **(14 pts)** Consider points $P(1, -1, 2)$, $Q(-2, 0, -1)$ and $R(0, -2, 1)$

   a) Find the area of the triangle determined by the points *P*,Q and R.

   b) Write the equation of the plane including these three points.

4. **(14pts)** Find the distance between the point $P(1, -3, 0)$ and the line $x = 2t,\ y = 2,\ z = 1 - t$.

5. **(16pts)** Sketch each curve (it may help to express *y*(*t*) as a function of *x*(*t*)), and find the velocity vector at time zero for each curve.

   a) $\mathbf{r}(t) = \langle e^{2t}, 1 - e^{4t} \rangle$ $(0 \le t < \infty)$

   b) $\mathbf{r}(t) = \langle 3\cos t, 2\cos t - 3 \rangle$ $(-\pi/2 \le t \le \pi/2)$

6. **(14pts)** Classify the following quadric surface (if you can't recall the name, you have to sketch it), and classify all of its cross-sections:

$$4z^2 + 2x^2 - 12x - y^2 + 20 = 0$$

7. **(14pts)** Show that the range of an object launched at a speed v_o and launch angle α from the Earth's surface equals $R = \frac{v_0^2}{g}\sin 2\alpha$, where *g* is the acceleration of free fall. At what angle is the maximal range achieved?