

Read each problem carefully. Show all work for each problem. No electronic devices or notes allowed

1. (16 pts) Find the limit, or show that it does not exist:

a.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^3}{x^4 + y^4}$     b.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^3}{x^3 + y^3}$     c.  $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 y)}{x}$

2. (18 pts) For the function  $f(x, y) = \sqrt{1 + 2x + y^2}$

- Find the domain and the range of this function
- Sketch several level curves of this function, indicating the function value for each
- Find the linearization of the function at point (1,1), and use it to estimate  $f(1.2, 0.9)$
- Use the quadratic approximation to find a better estimate for  $f(1.2, 0.9)$

3. (18 pts) Consider the surface  $F(x, y, z) = 5x^3 - y^2 - 3xye^z + z = 1$

- Find the plane tangent to this surface at point (1, 1, 0)
- Use implicit differentiation to find  $\frac{\partial z}{\partial x}(1,1)$ ,  $\frac{\partial z}{\partial y}(1,1)$ , and  $\frac{\partial^2 z}{\partial y^2}(1,1)$

4. (16 pts) Find the local maxima, minima and saddle points of

$$f(x, y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y.$$

5. (16 pts) Find the extreme values of the function  $f(x, y, z) = xy + z^2$  on the curve of intersection of the plane  $z = x + y$  and the sphere  $x^2 + y^2 + z^2 = 4$

6. (16 pts) Sketch the region of integration, reverse the order of integration, and evaluate the integral:

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx$$