

Read each problem carefully. Show all work for each problem. No calculators or notes.

1. **(16 pts)** Use polar coordinates to find the area lying outside the unit circle and inside a single petal of the rose given by  $r = 2 \cos 3\theta$
2. **(20 pts)** Use an appropriate coordinate transformation (i.e. a multiple substitution) to find the area in the 1<sup>st</sup> quadrant enclosed by parabolas  $y=x^2$  and  $y=2x^2$  and by hyperbolas  $xy=1$  and  $xy=4$ .
3. **(22 pts)** Sketch and find the mass of the solid of density  $\delta=z$  enclosed in the upper half-plane by the cylinder  $x^2 + y^2 = 1$ , the cone  $z^2 = x^2 + y^2$ , and the  $xy$ -plane, using two different coordinate systems:
  - a) Cylindrical coordinates
  - b) Spherical coordinates.
4. **(18pts)** Sketch the vector field  $\vec{F}(x, y) = \langle y^2, x^2, 0 \rangle$  in the  $x$ - $y$  plane, and calculate its curl. Find the set of points where the curl equals zero, and use your sketch to explain why the curl should be expected to be zero at these points.
5. **(24 pts)** Consider vector field  $\vec{F}(x, y) = \langle 2xy + z \cos(xz), x^2, x \cos(xz) + 1 \rangle$ .
  - a) Show that  $\vec{F}$  is conservative
  - b) Find the potential function of this vector field.
  - c) Use curve parameterization to directly calculate its line integral along the path in the figure, where  $C_1$  is a parabola from the origin to point  $(1,0,1)$  and  $C_2$  is a straight line from  $(1,0,1)$  to  $(1,1,1)$ .
  - d) Compare the result in part (c) to the line integral value obtained using the potential function

