Math 213 EXAM III April 23, 2014

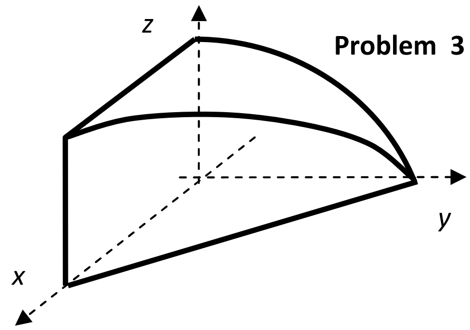
Read each problem carefully. Show all work for each problem. No calculators or notes.

1. **(14pts)** Sketch the region of integration, change the order of integration, and evaluate the integral:

$$\int_0^3 \int_{y^2}^9 y\cos\left(\frac{\pi x^2}{2}\right) dx\, dy$$

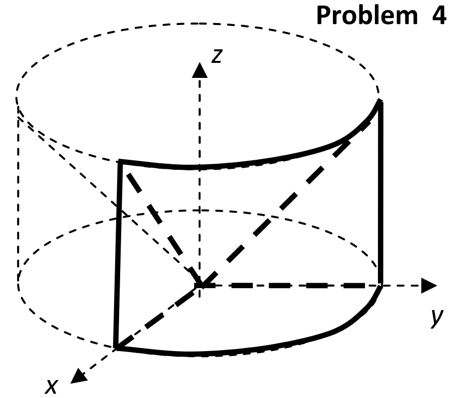
2. **(14pts)** Sketch and use double integration in polar coordinates to find the area enclosed inside the curve $r = 3 + 2\cos\theta$ and outside the circle r=4.

Problem 3



3. **(14pts)** Set up the limits on the triple integral over the region enclosed in the first octant by the surfaces $x + y = 4$ and $y^2 + 4z^2 = 16$ (see figure), in any three different integration orders. Do not calculate the integral. Which orders of integration are not possible without breaking the integral into several pieces? (recall that there are a total of six possible integration orders for a triple integral)

Problem 4

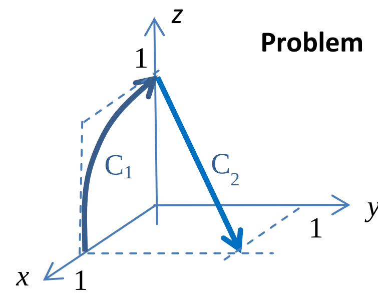


4. **(18pts)** Find the mass of an object with volume density $\delta(x, y, x) = xz$ enclosed **in the first octant** inside the cylinder $x^2 + y^2 = 4$ and below the cone $z^2 = x^2 + y^2$ (see figure), using triple integration in two different coordinate systems: (a) cylindrical (b) spherical.

5. **(14pts)** Use coordinate transformation *u*=2*x*–*y*, *v*=*y* to calculate the following integral:

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3 (2x - y) e^{(2x-y)^2}\, dx\, dy$$

Problem 6



6. **(16pts)** Sketch the vector force field $\vec{F}(\vec{r}) = \langle y,\ y - x,\ 0\rangle$ in the *x-y* plane, and calculate its curl and its divergence. Then, find work performed by this force over a piece-wise smooth curve consisting of a unit quarter-circle in the *x-z* plane, C_1, and a straight line C_2, as indicated in the figure.

7. **(14pts)** Consider vector field

$$\vec{\mathbf{F}}(x, y, z) = \left\langle y\sin(xy),\ \ 2(z+1)^{3/2} + x\sin(xy) + z^3,\ \ 3y\sqrt{z+1} + 3z^2 y\right\rangle$$

a) Find the potential function of this vector field.

b) Find work done by this force over a linear trajectory from point (1, 0, 1) to point (0, 1, 3)