Math 213 • Common Exam 1
February 12, 2014

Show all work for each problem, clearly explaining your solution. This is a closed-book exam: no notes or electronic devices allowed. **Note that points add up to 112: you may skip one short problem**

1. **(11pts)** Find the projection of vector $\mathbf{u} = \langle 1, 0, 2 \rangle = \mathbf{j} + 2\mathbf{k}$ onto vector $\mathbf{v} = \langle 1, 1, 3 \rangle = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$

2. **(12pts)** Consider lines $\mathbf{r}_1(t) = \langle 0, 2, 1 \rangle + t \langle 1, -1, 1 \rangle$, and $\mathbf{r}_2(t) = \langle 2, 6, 3 \rangle + t \langle 2, 1, 5 \rangle$. Are these lines parallel, intersecting, or neither (i.e., skew)?

3. **(14pts)** Consider vectors $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{j}$.

   a) Find the area of the parallelogram with sides formed by vectors $\mathbf{u}$ and $\mathbf{v}$

   b) Find the angle between vectors $\mathbf{u}$ and $\mathbf{v}$

4. **(12pts)** Find the equation of the line of intersection of the planes $x + 2y - z = 3$ and $x - 3y + 5z = 1$

5. **(12pts)** Find the equation of the plane containing points (0, 1, 0), (1, 0,−2) and (2, 3, 0).

6. **(15pts)** Consider the shortest distance between some point *P* and the line passing through point *S* and parallel to vector **v**.

   a) Which of the following formulas gives the correct expression for this distance:

   $$(i)\ D = \frac{|\overrightarrow{SP} \times \vec{\mathbf{v}}|}{|\vec{\mathbf{v}}|} \qquad (ii)\ D = \frac{|\overrightarrow{SP} \cdot \vec{\mathbf{v}}|}{|\vec{\mathbf{v}}|} \qquad (iii)\ D = \frac{|\overrightarrow{SP} \cdot \vec{\mathbf{v}}|}{|\overrightarrow{SP}|} \qquad (iv)\ D = \frac{|\overrightarrow{SP} \times \vec{\mathbf{v}}|}{|\overrightarrow{SP}|}$$

   b) Explain the distance formula using a simple sketch (the sketch should show vectors $\overrightarrow{SP}$ and $\vec{\mathbf{v}}$, the angle between them, and the distance between the line and the point).

   c) Find the distance between the point (1, 1, 1) and the line $x = t,\ y = 1 - t,\ z = 3$.

7. **(12pts)** Consider the surface $y - z^2 - 6z - x^2 + 13 = 0$. Categorize and sketch a couple different x-sections and y-sections, and categorize the surface. Make a rough sketch (make sure to label the axes).

8. **(12pts)** Describe geometrically the position of all points P satisfying the equation $\overrightarrow{P_0P} \times \vec{\mathbf{v}} = 0$, where P_0 is a fixed point, and $\overrightarrow{P_0P}$ is a vector from P_0 to P. Explain your answer.

9. **(12pts)** Which of the following expressions are **always** true? Explain (very briefly)

   (a) $|\vec{\mathbf{v}} \times \vec{\mathbf{v}}| = |\vec{\mathbf{v}}|^2$ (b) $|\vec{\mathbf{v}} \times \vec{\mathbf{v}}| = 0$ (c) $|\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}| = 0$ (d) $\vec{\mathbf{v}} \cdot (\vec{\mathbf{u}} \times \vec{\mathbf{v}}) = 0$ (e) $\left|\vec{\mathbf{v}} \cdot (\vec{\mathbf{u}} \times \vec{\mathbf{w}})\right| = \left|\vec{\mathbf{w}} \cdot (\vec{\mathbf{u}} \times \vec{\mathbf{v}})\right|$