Math 213 ● Final exam ● May 13, 2014

1. **(10 pts)** Consider an object moving with acceleration $a(t) = \left\langle 3\cos^2 t \sin t,\ \dfrac{t^3}{t^2+1} \right\rangle$. Find its velocity if the initial velocity is $\mathbf{v}(0) = \langle 0,\ 2 \rangle$.

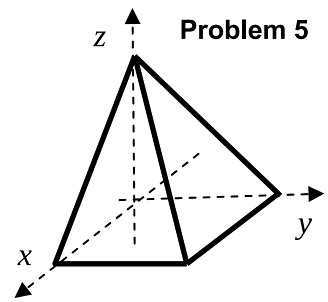
2. **(10 pts)** Find the limit, or show that it does not exist: **(a)** $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x+y}$ **(b)** $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{\sin(x+y)}$

3. **(14 pts)** Consider the scalar field $f(x,y) = \sqrt{6 - 4x - y^2}$. Find the domain and the range of this field and sketch the domain along with any two of its level curves. Then, use the linear approximation to estimate $f(1.04,\ 0.98)$. Finally, find the rate of change of this function at point (1,1) in the direction of vector $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$

4. **(12 pts)** Find local maxima, minima and saddle points of function $f = e^x\left(x^2 - y^2\right)$

5. **(14 pts)** Find the mass of the solid of density δ=*y* enclosed in the first octant by the planes $x + z = 1$ and $y + z = 1$, as shown in the figure. Which orders of integration *cannot* be used without breaking apart the integral into several pieces?

Problem 5



6. **(14 pts)** Sketch the vector field $\vec{\mathbf{F}}(x,y,z) = \langle 0,\ x \rangle$, and verify the Green's Theorem in circulation-curl form for this vector field and the region between the curves $x + 2y = 0$ and $x = 3 - y^2$, by calculating both integrals that appear in this theorem. Make sure to sketch the region of integration

7. **(10 pts)** Find the total area of the conical surface $z^2 = x^2 + y^2$ enclosed between the planes $z = 0$ and $z = 4$, using surface parametrization in polar variables *r* and θ.

8. **(16 pts)** Verify the Divergence Theorem for the vector field $\vec{\mathbf{F}} = \langle 0,\ 0,\ z \rangle$ and the region enclosed between the surfaces $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$, by calculating the flux of $\vec{\mathbf{F}}$ across the boundary of this region, in the outward direction, and comparing the result with the volume integral of the divergence of $\vec{\mathbf{F}}$. Make sure to sketch the region of integration.

Extra credit (6 pts) Calculate the following derivatives, where $\vec{\mathbf{r}} = \langle x, y, z \rangle$ denotes the position vector, and $\rho = |\vec{\mathbf{r}}| = \sqrt{x^2 + y^2 + z^2}$ denotes its length (distance from the origin). Simplify your results, expressing them in terms of variables $\vec{\mathbf{r}}$ and ρ only (instead of *x*, *y* and *z*)

a) $\vec{\nabla}(\ln \rho)$ **b)** $\vec{\nabla} \cdot \left(\dfrac{\vec{\mathbf{r}}}{\rho^2} \right)$

Some useful formulas (in random order):

$$\oint_{\partial R} \mathbf{F} \cdot \mathbf{n}\, ds = \iint_{R} \nabla \cdot \mathbf{F}\, dA$$

$$\oint_{\partial R} \left(M\, dy - N\, dx\right) = \iint_{R} \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\, dy$$

$$\oint_{\partial S} \mathbf{F} \cdot \mathbf{dr} = \iint_{S} \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma$$

$$\oiint_{\partial V} \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_{V} \nabla \cdot \mathbf{F}\; dV$$

$$\oint_{\partial R} \left(M\, dx + N\, dy\right) = \iint_{R} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$

$$\oint_{\partial R} \mathbf{F} \cdot \mathbf{dr} = \iint_{R} \nabla \times \mathbf{F} \cdot \mathbf{k}\, dA$$