

Read each problem carefully. Show all work for each problem. No electronic devices or notes allowed

1. Find the limit, or show that it does not exist:

a. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^3}{(x^2 + y^2)^2}$ b. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^4 + y^4}$ c. $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 y)}{x}$

2. For the function $f(x, y) = \sqrt{1 + 2x + y^2}$

- a. Find the linearization of the function at point (1,1), and use it to estimate $f(1.2, 0.9)$.
b. Find the derivative of this function at point (1,1) in the direction of vector $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$

3. Use the chain rule to Evaluate $\frac{\partial w}{\partial u}$ at $u=1, v=1$ for $w = x \cos(y e^x)$

where $x = uv, y = u - v$

4. Consider the surface $F(x, y, z) = xye^{2z} + z = 1$

- a. Find the plane tangent to this surface at point (1, 1, 0)
b. Use implicit differentiation to find $\frac{\partial z}{\partial x}(1,1)$ and $\frac{\partial z}{\partial y}(1,1)$

5. Find the local maxima, minima and saddle points of

$$f(x, y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y.$$