1. Verify the conclusions of the Green's Theorem, both in the circulation-curl form and in the flux-divergence form, for the vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = \langle 1,\ x^2 \rangle$, and the region enclosed between the curves $y = x^2 - x$ and $y = x$. Start by sketching the region of integration.

Green's Theorem, circulation-curl (tangential) form:

$$\oint_{\partial R} \mathbf{F} \cdot \mathbf{dr} = \iint_R \nabla \times \mathbf{F} \cdot \mathbf{k}\, dA \ \Rightarrow\ \oint_{\partial R} (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

Green's Theorem, flux-divergence (normal) form:

$$\oint_{\partial R} \mathbf{F} \cdot \mathbf{n}\, ds = \iint_R \nabla \cdot \mathbf{F}\, dA \ \Rightarrow\ \oint_{\partial R} (M\,dy - N\,dx) = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx\,dy$$

2. Verify the conclusions of the Stokes Theorem for the vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = \langle 0,\ x^2,\ z \rangle$ and the part of the surface $z = 1 - x - y^2$ enclosed in the first octant, with the normal oriented away from the origin. Start by sketching this surface.

Stokes Theorem: $\displaystyle \oint_{\partial S} \mathbf{F} \cdot \mathbf{dr} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma$

3. Verify the conclusions of the Divergence Theorem for the volume enclosed by the surface $z + x^2 + y^2 = 4$ above the x-y plane, for the vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = \langle x,\ y,\ 1 + z \rangle$. Start by sketching the volume of integration.

Divergence Theorem: $\displaystyle \oiint_{\partial V} \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_V \nabla \cdot \mathbf{F}\ dV$