Read each problem carefully. Show all work for each problem. No electronic devices or notes allowed

1. **(12 pts)** A projectile is launched from ground level at an angle $\theta=\pi/4$, hitting a target 1000m away, also at ground level. What is the initial speed of the projectile? Use the following approximate value for the acceleration of free fall: $g \approx 10$ m/s^2.

2. **(12 pts)** Consider an object moving with acceleration $\mathbf{a}(t) = \left\langle \frac{1}{\sqrt{t+1}},\ e^{2t} \right\rangle$. Find its velocity and position if its initial velocity is $\mathbf{v}(0) = \left\langle 1,\ \frac{1}{2} \right\rangle$ and its initial position is $\mathbf{r}(0) = \left\langle \frac{1}{3},\ 1 \right\rangle$

3. **(12 pts)** Calculate the length of this curve (hint: when taking the integral, factor out *t* from the integrand, and make a simple substitution):

$$\mathbf{r}(t) = \left\langle t^3,\ 1+t^2 \right\rangle,\ t \in [\,0,\ 1]$$

Extra credit (5 pts): Sketch this vector function on the interval $t \in [-2,\ 2]$ as a trajectory in the (x,y) plane.

4. **(15 pts)** Find the limit, or show that it does not exist

a. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ b. $\lim\limits_{(x,y)\to(1,-1)} \frac{x^4 - x^2y^2}{x^2 + xy}$ **c.** $\lim\limits_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{x^2y^2}$

5. **(25 pts)** For the function $f(x,y) = \ln\left(x - y^3\right)$

   a. **(5pts)** Find the domain and the range of this function. Sketch the domain as a region in the (*x*, *y*) plane. Is it an open or a closed region?

   b. **(4pts)** Sketch the following level curves: *f*=0, *f*=1

   c. **(8pts)** Find the linearization of the function at point (2, 1), and use it to estimate *f*(1.8, 1.1)

   d. **(4pts)** Find the rate of change of *f*(*x*,*y*) at point (2, 1) in the direction of vector $-2\mathbf{i} + 3\mathbf{j}$

   e. **(4pts)** Find the unit vector in the direction of zero change of *f*(*x*,*y*) at point (2, 1)

6. **(12 pts)** Use the chain rule to evaluate $\frac{\partial f}{\partial t}$ at *s*=1, *t*=1 where

$$f(x,y) = 3x + y + e^{\sin\left(y^2\right)/x},\ x = s/t,\ y = s - t$$

7. **(12 pts)** Find the local maxima, minima and saddle points of

$$f(x,y) = xy + 2x - \ln\left(x^2 y\right)$$