$\lambda_0 = 0\ , \ \phi_0(x) = 1$

$\lambda_n = \frac{n^2\pi^2}{L^2}\ , \ \phi_n(x) = \cos\left(\frac{n\pi x}{L}\right) \quad n = 1, 2, \ldots$

and $\quad h(t) \to h_n(t) = e^{-\left(1+\frac{n^2\pi^2}{L^2}\right)t} \ ; n = 0, 1, 2, \ldots$ so

the general solution is

10 $u(x,t) = e^{-t} A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) e^{-\left(1+\frac{n^2\pi^2}{L^2}\right)t}$ where

note!

$A_0 = \frac{1}{L}\int_0^L f(x)\,dx$ and $A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

to satisfy the initial condition.

Also, $u(x,t) \underset{t\to\infty}{\longrightarrow} 0$ although we have no-flux boundary conditions.

5 That's ok since although heat energy can't escape out the ends at $x = 0, L$ we have a <u>sink</u> of heat energy $(-u)$ in the problem.