Math 331 · Midterm Exam

October 15, 2008

This is a closed-book test. Neither notes nor calculators are to be used. Check your answers

You can choose between problem 1a and problem 1b (no extra credit for doing both)

(1a, 15pts) The following equation describes the conservation of energy in a thin rod:

$$\frac{\partial e}{\partial t} = \pm \frac{\partial \varphi}{\partial x}$$

Here $e(x,t)$ is the energy density. Derive this equation, and give the correct value of the sign of the right-hand side; sketch a simple picture to explain the derivation. What is the meaning and the physical units of function $\varphi(x,t)$?

(1b, 15pts) Solve this ODE to find $y(x)$ [hint: we solved this type of ODE in class]

$$\begin{cases} x^2 \dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} - 3y = 0 \\ y(1) = y'(1) = 0 \end{cases}$$

(2, 15pts) Separate the variables in the following partial differential equation for $u(x,y)$ and write down the resulting two ODEs (Do not solve).

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y} = 0$$

(**3, 15pts)** Find the sine series for the function $f(x)=x$ on the interval [0, 1] ($L=1$). Write down the first three non-zero terms. Is this sine series continuous? Explain the answer about the continuity using a sketch

(4, 15pts) Find the **equilibrium** solution (*do not* calculate the full time-dependent solution). [Note the third power of r]

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{k}{r^3}\dfrac{\partial}{\partial r}\left(r^3 \dfrac{\partial u}{\partial r}\right), \quad 1 \le r \le 2 \\ u(1,t) = 0, \quad u(2,t) = 3 \\ u(r,0) = f(r) \end{cases}$$

(5, 40pts) Solve the Laplace's equation ($u_{xx}+ u_{yy}=0$) with the given boundary conditions inside a rectangle, $0 \le x \le L,\ \ 0 \le y \le H$. **Show all the steps** in your solution, but you don't have to explain each step in a lot of detail. Check your answer.

$$\begin{cases} \dfrac{\partial u}{\partial y}(x,0) = 0, \quad u(x,H) = 0 \\ u(0,y) = u(L,y) = 4\cos\dfrac{\pi y}{2H} \end{cases}$$