MATH 331-001 • Midterm Examination • Victor Matveev • October 26, 2016

1. **(15pts, 22min)** Suppose $f(x)$ equals $\sin(\pi x)$ on the interval $0 < x < 1$ (***L*=1**), and is an **even periodic extension** of this function to the rest of the real line. Find the **cosine** series for this even periodic extension. Make a rough sketch of this periodic function, as well as the sum of the first two non-zero Fourier terms (i.e. graph the sum of the constant plus the next non-zero term), for $-2<x<2$.

   Hint: $2\sin\alpha\cos\beta = \sin(\alpha+\beta)+\sin(\alpha-\beta)$

2. **(20pts, 14min)** Consider a rod/cable of length L=1 with constant thermal properties ($c\rho$ = 1, K_o=1):

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + \pi^2 \cos\dfrac{\pi x}{2} \quad (0<x<1,\ t>0) \\ u_x(0,t) = u(0,t) \\ u_x(1,t) = 0 \end{cases}$$

   a) Determine the equilibrium temperature distribution (note: you don't have to compute total energy)

   b) Make a rough sketch of the equilibrium temperature distribution (note: you don't have to solve part "a" to produce a rough sketch). You may assume that the temperature is positive.

   c) Explain the equilibrium: where does the energy enter the rod, and where does it leave the rod?

3. **(15pts, 10min)** Separate variables in the following differential equation, and simplify the two equations you obtain (i.e. get rid of any quotients in your equations). Do ***not*** solve the equations!

$$r\frac{\partial^2 u}{\partial t^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) - r\frac{\partial u}{\partial t}$$

4. **(30pts, 32min)** Use separation of variables to solve the following wave equation problem. You do not have to provide more detail than you need, but make sure to indicate briefly all the main steps. When separating variables, recall that your solution will be simpler if you move the constant c^2 away from the boundary value part of the problem. Make sure to check your solution against the given equation, the boundary conditions, and the initial conditions.

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = c^2\dfrac{\partial^2 u}{\partial x^2} \quad (0<x<L,\ t>0) \\ u(0,t) = u(L,t) = 0 \\ u(x,0) = 0;\ \dfrac{\partial u}{\partial t}(x,0) = 1 \end{cases}$$

5. **(10pts, 6min)** Answer **one** of the following two questions (i.e. complete part "a" or part "b"):

   a) In the energy conservation equation $\dfrac{\partial e}{\partial t} = -\dfrac{\partial \phi}{\partial x}$, write down the physical units of e and ϕ

   b) In the wave equation of a string, $\rho_o u_{tt} = T_o u_{xx}$, write down the physical units of ρ_o and T_o

6. **(10pts, 6min)** Which of the following function(s) do/does ***not*** agree with the Laplace's equation, $u_{xx} + u_{yy} = 0$? Explain very briefly (Hint: examine the curvatures with respect to x and y)