Math 331-001 • Midterm Examination • Victor Matveev • Fall 2017

Please read the assignment carefully, and show all work. No notes or electronic devices allowed.

1. **(20pts, 20min)** Consider a function defined as $f(x)=\begin{cases}0, & 0<x<\dfrac{1}{2}\\ 1, & \dfrac{1}{2}<x<1\end{cases}$ on the interval $0<x<1$.

   a) Graph the even periodic extension of this function to the interval [-3, 3] (assume ***L*=1**), and the sum of 1st two non-zero terms in the cosine series. Use the plot to guess the sign of coefficient A_1.

   b) Find the cosine series of this function, and write down the sum of first three non-zero terms.

2. **(20pts, 15min)** Consider a rod/cable of length *L*=1 with constant thermal properties:

$$\begin{cases}u_t(x,t)=u_{xx}(x,t)+\dfrac{3}{\sqrt{x}} & (0<x<1,\ t>0)\\ u_x(0,t)-u(0,t)=0\\ u_x(1,t)=1\end{cases}$$

   a) Find the ***equilibrium*** solution. Make sure to check your answer.

   b) Make a rough plot of two functions: the source term and the equilibrium.

   c) Calculate the heat flux at the endpoints (at equilibrium); you may set K_o=1.

   d) Explain the equilibrium: where does the energy enter the rod, and where does it leave the rod?

3. **(16pts, 10min)** Separate variables in the following differential equation (plug in $u(x,y)=g(x)h(y)$), and write down the two equations you obtain. Do ***not*** solve!

$$y\frac{\partial}{\partial x}\left(x\frac{\partial u}{\partial x}\right)+x\frac{\partial u}{\partial y}+\frac{x}{y}\frac{\partial^2 u}{\partial y^2}=0$$

4. **(28pts, 25min)** Solve the following partial differential equation inside a rectangle ($0<x<L$, $0<y<H$). *Make sure to explain **all** steps.* When solving the homogeneous boundary value problem, sketch any two solutions (e.g. ϕ_1 and ϕ_2). **Check** your solution, in particular the boundary conditions.

$$\begin{cases}u_{xx}+u_{yy}=0 & (0<x<L,\ \ 0<y<H)\\ u(0,y)=g(y);\ \ u_x(L,y)=0\\ u_y(x,0)=0;\ \ u(x,H)=0\end{cases}$$

5. **(16pts, 10min)** Consider (but do not solve) the wave equation for *u*(*x*,*t*):

$$\begin{cases}u_{tt}=c^2u_{xx} & \begin{pmatrix}0<x<1\\ t>0\end{pmatrix}\\ u_x(0,t)=u_x(1,t)=0\\ u(x,0)=0\\ u_t(x,0)=2\end{cases}$$

   For each of the following functions, check whether they satisfy the boundary conditions, initial conditions, and the partial differential equation in the problem above.

$$(a)\ \ u(x,t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(n\pi x)\sin(cn\pi t) \qquad (b)\ \ u(x,t)=2t$$

$$(c)\ \ u(x,t)=2\cos(\pi x)\sinh(c\pi t) \qquad (d)\ u(x,t)=2\cos(\pi x)\cos(c\pi t)$$