Math 331-002 • Midterm Examination • Victor Matveev • March 9, 2017

Please read the assignment carefully, and show all work. No notes or electronic devices allowed.

1. **(14pts, 20min)** Consider a function defined as $f(x) = 1 - x$ for $0 < x < 1$ (assume L=1).
   a) Sketch the even periodic extension of this function to the interval [-3, 3]
   b) Find the cosine series of this function, and write down the first three non-zero terms.
   c) Sketch the sum of first two non-zero terms in the series.

2. **(20pts, 15min)** Consider a rod/cable of length L=1 with constant thermal properties:

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + \dfrac{4}{(1+x)^2} \quad (0 < x < 1,\ t > 0) \\ u(0,t) = 0 \\ u_x(1,t) + u(1,t) = 0 \end{cases}$$

   a) Find the ***equilibrium*** solution, $u_{eq}(x)$. Make sure to check your answer.
   b) Sketch the source term and the equilibrium $u_{eq}(x)$ (you can do this before you solve for $u_{eq}(x)$)
   c) Explain the equilibrium: where does the energy enter the rod, and where does it leave the rod?

3. **(12pts, 7min)** Separate variables in the following differential equation (plug in $u(x,y) = g(x)h(y)$), write down the two equations you obtain, and simplify them slightly by writing them in a form that does not contain quotients. Do ***not*** solve!

$$\frac{x}{y}\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} + \frac{1}{xy}\frac{\partial u}{\partial x} = 0$$

4. **(30pts, 25min)** Solve the following partial differential equation inside a square $(0 < x < 1,\ 0 < y < 1)$. Make sure to indicate *briefly* the main steps. When solving the homogeneous boundary value problem, you don't have to consider *all* signs of λ, but do check λ=0, and sketch any two eigenfunctions (e.g. ϕ_1 and ϕ_2). Make sure to check your solution.

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0 \qquad (0 < x < 1,\ 0 < y < 1) \\ \dfrac{\partial u}{\partial x}(0,y) = 1;\ u(1,y) = 0 \\ u(x,0) = \dfrac{\partial u}{\partial y}(x,1) = 0 \end{cases}$$

5. **(12pts, 10min)** Write down the general solution to the following *ordinary* differential equations. Hint: at least one of these equations is an ***equidimensional (Euler)*** ordinary differential equation which we encountered when solving the Laplace's equation in a disk.

   a) $x^2\dfrac{d^2g}{dx^2} + 2x\dfrac{dg}{dx} - 2g(x) = 0$ b) $\dfrac{d^2g}{dx^2} + 5\dfrac{dg}{dx} + 6g(x) = 0$

One last problem on the reverse side…

6. **(12pts, 8min)** Which of the following function surfaces do *not* satisfiy the partial differential equation written below? For each answer (each surface) you choose, identify at least ***one*** condition which is not satisfied. ***Pay particular attention to given boundary conditions***, axis labels and coordinates.

$$\begin{cases} \dfrac{\partial^2 u}{\partial y^2} = -\dfrac{\partial^2 u}{\partial x^2} \quad \text{where } u = u(x,y) \text{ with } (0 < x < 1,\ \ 0 < y < 0.5) \\ u(0,y) = u(1,y) = 0 \\ u(x,0) = u(x,0.5) = \alpha \sin(\pi x) \quad (\text{where } \alpha = const) \end{cases}$$