Math 335-002 * Midterm examination

February 20, 2008

This is a closed-book exam: notes or calculators are *not* allowed. Please show all solution steps to receive full credit.

1. (10) Write an equation of plane that contains points (1, 1, 1), (1, 2, 3) and (3, 2, 1) [Hint: first, find a vector perpendicular to this plane, using vector algebra]

2. (12) Find the divergence of the vector field $\vec{\mathbf{V}}(\vec{\mathbf{r}}) = \ln(r)\,\vec{\mathbf{r}}$, where r is the length of the position vector ($r = |\vec{\mathbf{r}}|$). Simplify the answer (i.e. express it as a function of r and/or $\vec{\mathbf{r}}$ only).

3. (18) Consider the following vector expression: $\vec{\mathbf{b}} \times \vec{\mathbf{a}} \bullet \vec{\mathbf{c}} \times \vec{\mathbf{b}}$

a) Re-write this vector expression in suffix notation (do not simplify)

b) Get rid of all cross products in this expression, using vector algebra

4. (15) Consider a scalar field $f(x, y) = \sqrt{\ln y + x}$.

a) Use the linear approximation to estimate $f(1.1, 1.2)$.

b) Draw isocurves f=0, f=1, f=2

5. (15) Sketch the vector field $\vec{\mathbf{u}} = (y-x, x, 0)$. Is this vector field conservative? If yes, find its potential function.

6. (10) Simplify and convert into vector form: $a_l\, a_q\, a_m\, b_n\, \varepsilon_{kmp}\, \delta_{kj}\, \delta_{pn}\, \delta_{lq}$

7. (22) Consider a vector field $\vec{\mathbf{u}}(\vec{\mathbf{r}}) = \left(e^y, e^{2y}, x^2 + z^2\right)$.

a) Re-write the following quantities using symbols *grad*, *div*, *curl* and ∇^2, and compute them: $\left(\vec{\nabla} \bullet \vec{\nabla}\right) \vec{\mathbf{u}}$, $\vec{\nabla}\left(\vec{\nabla} \bullet \vec{\mathbf{u}}\right)$, $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\mathbf{u}}\right)$, $\vec{\nabla} \bullet \vec{\nabla} \times \vec{\mathbf{u}}$

b) Which of the quantities in part "a" are linearly dependent? Write the relationship between these quantities.

Alternative problem **2** (worth **8** points instead of **12**): Calculate the gradient of ln(r).