Math 335-002 * Midterm #2
March 26, 2008

Please show all work to receive full credit. Notes and calculators are *not* allowed

1. (20pts) Use ***suffix notation*** to expand or simplify the following expressions, and convert the result into vector form:
   a) $\vec{\nabla}\cdot(\vec{\mathbf{r}}\times\vec{\mathbf{u}})$, where $\vec{\mathbf{r}}$ is the position vector, and $\vec{\mathbf{u}}$ is an arbitrary vector field
   b) $\left(\left(\left(\vec{\mathbf{b}}\times\vec{\mathbf{a}}\right)\times\vec{\mathbf{a}}\right)\times\vec{\mathbf{a}}\right)$

2. (20pts) Calculate the work done by the force $\vec{\mathbf{F}}=(e^x, \sqrt{y}, 0)$ along the following two paths connecting points A=(2,0,0) and B=(0,2,0):
   a) A straight line
   b) A circular arc around the origin of radius 2
   c) Is this a conservative vector field? If yes, check your integration by finding the potential function.

3. (24pts) Verify the divergence theorem (by calculating the appropriate surface and volume integrals) for the field $\vec{\mathbf{u}}=(x, 0, 0)$, with the volume in the 1st octant defined by $x + 2y + 2z \le 2$. Start your solution by sketching this surface (hint: find the intersection of the boundary $x + 2y + 2z = 2$ with the three coordinate planes).

4. (20pts) Find the mass of an object enclosed between the surfaces $x^2 - y^2 + z^2 + 1 = 0$, $x=0$ and $y=2$, with the mass density $\rho(\vec{\mathbf{r}}) = y^2 + 1$. Sketch this object [Hint: find the simplest cross-section]

5. (16pts) Which of the following integrals are always zero for ***any*** differentiable fields $\vec{\mathbf{u}}$ or f? [Hint: the divergence theorem will help with some of these integrals].

   a) $\oiint_S \vec{\nabla}\times\vec{\mathbf{u}}\cdot\vec{\mathbf{n}}\,dS$ b) $\oiint_S \vec{\nabla}f\cdot\vec{\mathbf{n}}\,dS$ c) $\oint_C \vec{\nabla}f\cdot d\vec{\mathbf{r}}$ d) $\iiint_V \vec{\nabla}\times\vec{\nabla}f\,dV$