Math 335-002 * Midterm examination

February 14, 2007

This is a closed-book exam: notes or calculators are *not* allowed. Please show all solution steps to receive full credit.

1. (10) Use vector algebra to find the angle between vectors (2, 0, 2) and (0, 2, 2). What is the area of the parallelogram whose sides are formed by these two vectors?

2. (10) Simplify the following expression: $(\vec{c} - \vec{b}) \cdot (\vec{c} \times \vec{b}) \times \vec{c}$

3. (10) In the physics of gravity and electrostatics the potential function (potential energy) has the form $\varphi(\vec{\mathbf{r}}) = c / r$, where c is a constant and r is the length of the position vector ($r = |\vec{\mathbf{r}}|$). Calculate the field $\vec{\mathbf{F}}$ (the force) for this potential $\varphi(\vec{\mathbf{r}})$, using the formula $\vec{\mathbf{F}} = -\vec{\nabla}\varphi$ (note the minus sign). Show that it can be written as $\vec{\mathbf{F}} = c\,\vec{\mathbf{r}} / r^n$, where n is an integer power you have to determine.

4. (20) Consider a scalar field $f(x, y) = \ln(x^3 / y)$
   a) Calculate $\vec{\nabla} f$
   b) Estimate $f(1.1, 1.2)$ using the linear approximation for f around an appropriately chosen nearby point $\mathbf{r}_0$
   c) Sketch this field and show the direction of increase of f on your sketch. Check by calculating the direction of increase at points (1, 1) and (-1, -1)

5. (15) Consider a vector field $\vec{\mathbf{u}} = (x - y,\ y - x,\ 0)$
   a) Sketch this field
   b) Calculate its divergence and curl, and explain your results in terms of your sketch.

6. (15) Simplify where possible and convert into vector form:

   a) $\delta_{kl}\, a_j\, \delta_{mj}\, a_k\, b_m$ b) $a_j\, c_m\, b_n\, b_k\, \varepsilon_{jmn}$ c) $\varepsilon_{ikm}\, \varepsilon_{mlj}\, a_k\, b_l\, \delta_{ij}$

7 (22) Consider a conservative vector field $\vec{\mathbf{V}}(\vec{\mathbf{r}}) = \left(xy^2,\ y(x^2 + z^2),\ zy^2\right)$
   a) Calculate the Laplacian of $\vec{\mathbf{V}}$
   b) Is this field irrotational, solenoidal, neither irrotational nor solenoidal, or both irrotational *and* solenoidal?
   c) Find the potential function of this field, $f(\vec{\mathbf{r}})$
   d) Calculate the Laplacian of f (hint: even if you failed to solve part *c*, you could still do this calculation)

Alternative to problem 4: if you somehow can't figure it out, solve all 4 parts (*a-d*) for a a somewhat different function $\boldsymbol{f(x, y) = \ln(y) / x}$**.** In part *c*, check the direction of increase using points (1, *e*) & (-1, *e*) (recall that $e \sim 2.718..$)