Math 335-002 * Midterm #2
March 21, 2007

Please show all work to receive full credit. Notes and calculators are *not* allowed

1. (18pts) Calculate the following derivatives **using suffix notation** (here $\vec{\mathbf{r}}$ is the position vector, and $r = |\vec{\mathbf{r}}|$):
   a) $\vec{\nabla} \cdot (\vec{\mathbf{r}} \sin r)$
   b) $\vec{\nabla} \times (\vec{\mathbf{a}} \times \vec{\mathbf{r}})$, where $\vec{\mathbf{a}}$ is any *constant* vector

2. (12pts) Use product rules to expand the expression $\vec{\nabla} \cdot \left[f\, \vec{\nabla} f + f\, \vec{\nabla} \times \vec{\mathbf{u}} \right]$, where $\vec{\mathbf{u}}$ is a vector field, and *f* is a scalar field satisfying the Laplace's equation, $\nabla^2 f = 0$. Simplify if possible.

3. (20pts) Calculate the line integrals (work) of the vector field (force) $\vec{\mathbf{F}} = (y^2, 2xy, 0)$ along two different paths connecting points A=(0,2,0) and B=(4,0,0):
   a) (8pts) A parabola y = $\sqrt{4-x}$ b) (8pts) A straight line
   c) (4pts) Explain how to obtain the answer to (a) and (b) without integration.

4. (20pts) Consider the part of the curved surface S given by $x^2 + y + z = 1$, enclosed within the region (octant) $x \geq 0$, $y \geq 0$, $z \geq 0$.

   a) (4pts) Sketch the intersections of this surface with each of the three coordinate planes bounding this surface on its three sides (x=0, y=0, and z=0)

   b) (16pts) Calculate $\iint_S \vec{\mathbf{u}} \cdot \vec{\mathbf{n}}\, dS$ for the field $\vec{\mathbf{u}} = \vec{\mathbf{r}} = (x, y, z)$, with $\vec{\mathbf{n}}$ pointing outward.

5. (20pts) Use the divergence theorem to calculate the surface integral $\iint_S \vec{\mathbf{u}} \cdot \vec{\mathbf{n}}\, dS$ of the field $\vec{\mathbf{u}} = (xz, yz, z^2)$ over the curved surface $x^2+y^2-z^2 \leq 0$, $0 \leq z \leq 1$, with $\vec{\mathbf{n}}$ pointing outward. Start your solution by sketching this surface (hint: you are supposed to convert the surface integration into volume integration).

6. (10pts) Use the divergence theorem to find the relationship between the volume of any object V and the integral of the position vector $\vec{\mathbf{r}}$ over the surface of this object, $\oiint_S \vec{\mathbf{r}} \cdot \vec{\mathbf{n}}\, dS$ (hint: simply apply the divergence theorem to this surface integral)

Alternative to problem 5 (13 points only) Calculate the mass of a solid of rotation $x^4 + y^2 + z^2 \leq 1$, $x \geq 0$, with mass density $\rho(\vec{\mathbf{r}}) = x^2$ (hint: it has a shape of a deformed hemisphere)