Math 335-002
Homework #10
Due date: March 5, 2008

1. Read p. 59 of the book, the derivation of the equation (3.20) on p. 58

2. Problems 2.1-2.3, p. 31

3. Find the line integral of the vector field $\vec{\mathbf{u}}=(x^2, y^{1/3}, z)$ along the curve given by $x=t^2$, $y=e^{3t}$, $z=e^{2t}$, for t varying from 0 to 1

4. Consider a conservative force $\boldsymbol{F}=-\nabla\varphi$ with a potential energy φ given by $\varphi=r^2$. Use line integration to calculate the work done by this force along the parabola $y=x^2$, for x varying from 0 to 1 (assume z=0). Compare this value with the difference in potential energy between the endpoints of the curve, φ(B)-φ(A).

5. Calculate the line integral of a vector field $\vec{\mathbf{u}}=(y^2, -x, 0)$ over the following curves connecting points A=(1,0,0) and B=(0,1,0):

   a. A horizontal line connecting point A and the origin (0,0,0) plus a vertical line connecting the origin and point B.
   b. A circular arc connecting points A and B (recall that trigonometric functions parametrize this circle)
   c. A straight line connecting points A and B

   Compare the three results. Is $\vec{\mathbf{u}}$ a conservative vector field? Calculate the curl of $\vec{\mathbf{u}}$ to check your conclusion.