Math 335-002
Homework #15
Due April 7

1. Sketch and use cylindrical coordinates to calculate the mass of a paraboloid cut in half, defined by $x^2 + y^2 \leq 1 - z$, $0 \leq z \leq 1$, $\boldsymbol{x \geq 0}$, with the mass density function given by $\rho = z\, x\sqrt{x^2 + y^2}$. Make sure you use correct limits for the angle φ.

2. According to both integral theorems, the integral of $\nabla \times \vec{\mathbf{u}}$ of any differentiable field $\vec{\mathbf{u}}$ over any **closed** surface is zero. *Use cylindrical coordinates* to verify this fact for the vector field $\vec{\mathbf{u}} = (xz, xz, 0)$, for the surface of the cylinder $x^2 + y^2 = 1$, $0 \leq z \leq 1$. Recall that the normal to the side surface is the unit vector $\vec{e}_R$.

3. Problem 6.5 on p. 113.

4. Problem 6.2 on page 107 – find the unit vectors, the scale factors (“stretch” factors), and the volume element, as we did in class for the cylindrical coordinate system, by differentiating the position vector with respect to u, v, and w. Note that x_1, x_2 and x_3 denote the Cartesian coordinates x, y, and z.