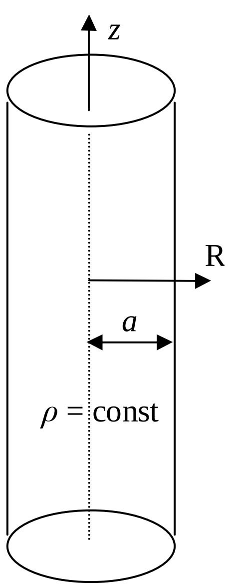
Math 335-002
Homework #19
Due April 21, 2007

1. Consider a sphere of radius a with charge density increasing with distance from the center as $\rho(r)=\alpha\, r$, where α is some constant. Find the electric potential Φ both inside and outside of the sphere, by integrating the Poisson's equation in spherical coordinates ($\Delta\Phi = -\rho/\varepsilon_0$ inside the sphere, and $\Delta\Phi = 0$ outside of the sphere), as we did in class. Assume that the solution depends on r only: $\Phi=\Phi(r)$. Note that the electric field should be continuous across the surface of the sphere; this condition will fix the integration constants

2. Check by differentiation that the potential inside and outside the sphere that you found in problem 1 satisfies the Poisson's equation re-written in Cartesian coordinates:

$$\Delta\Phi^{in} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi^{in} = -\frac{\alpha r}{\varepsilon_0} = -\frac{\alpha\sqrt{x^2+y^2+z^2}}{\varepsilon_0} \text{ - Inside the sphere}$$

$$\Delta\Phi^{out} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi^{out} = 0 \text{ - Outside the sphere}$$



3. Consider a very long cable (cylinder) of radius a with a constant charge density inside the cable, ρ, and zero charge density outside the cable. Use **cylindrical** coordinates to find the electric potential Φ and the radial component of electric field E_R both inside and outside of the cable, by solving (integrating) $\nabla^2\Phi = -\rho/\varepsilon_0$ inside the cable, and $\nabla^2\Phi = 0$ outside of the cable, as we did in class for a sphere in spherical coordinates. Assume that Φ depends only on R, the distance from the z-axis, which is the axis of the cable: $\Phi=\Phi(R)$. Use Eq. 6.16 for the Laplacian. Assume that E_R is continuous across the surface of the cable; this condition will fix one of the integration constant.

4. Use the divergence theorem instead of the Poisson's equation to find the electric field inside a uniformly charged sphere in example 8.3 on p. 136: $E_r(r) = \rho\, r / 3\,\varepsilon_0$. Hint: integrate both sides of equation $\nabla\bullet \mathrm{E} = \rho / \varepsilon_0$ over the volume of a sphere of radius $b<a$ to obtain $E_r(b) = \rho\, b / 3\,\varepsilon_0$