Math 335-002
Homework #22
Due date: April 30, 2008

1. If u_i is a vector, show that $\frac{\partial u_i}{\partial x_j}$ is a tensor of rank 2 (derive the transformation rule in terms of L_{ij}).

2. Find the conductivity tensor of a material (defined by $j_i = \sigma_{ik} E_k$) given the following 3 measurements of current density at different values of electric field:

   $\boldsymbol{j}$ = (0.8, 0.2, 0) A/m^2 when $\boldsymbol{E}$ = (2, 0, 0) V/m
   $\boldsymbol{j}$ = (0.7, 1.3, 0) A/m^2 when $\boldsymbol{E}$ = (1, 3, 0) V/m
   $\boldsymbol{j}$ = (0.9, 0.6, 0.5) A/m^2 when $\boldsymbol{E}$ = (2, 1, 1) V/m

   Hint: determine the first column of σ_{ik} using the 1st measurement, then use these results along with the 2nd measurement to determine the 2nd column, and so on. Units of σ_{ik} are (A/m^2)/(V/m)=A/(V·m)=S/m.

3. Show that the conductivity tensor you found in problem 2 will be diagonalized by a rotation around the *z*-axis by $\pi/4$. To do this, use matrix multiplication to find $\sigma' = L\,\sigma\,L^T$ (the matrix form of the rule $\sigma'_{ij} = L_{ik} L_{jl} \sigma_{kl}$), where a rotation around the z-axis is given by

   $$L_z(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Sketch or describe how the layers of the material are oriented with respect to the old and the new coordinate systems. Finally, use matrix multiplication to verify that LL^T=I

4. Problems 7.12, 7.13, 7.15 on p. 130