Math 335-002
Homework #3

Due date: January February 4 (not collected)

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Problem 1.16 on p. 20

2. Find the domain and range of the following 2D scalar fields, and sketch several isocurves for each field. Calculate the gradient and indicate the direction of the gradient along each isocurve:

   a) $F(x, y) = \ln(x^2 / y)$
   b) $G(x, y) = \cos(x - y)$

3. Problem 3.6, p. 53: Find the gradient of a 2D scalar field $f(\vec{r}) = \vec{r} \cdot \vec{a}$, where $\vec{a}$ is a constant vector, and $\vec{r} = (x, y)$ is the variable position vector. For $\vec{a} = (1, 2)$, sketch separately the scalar field (by showing its isocurves) and its gradient, which is a vector field. Comment on the relationship between the gradient and the vector $\vec{a}$

4. Problem 3.3 on p. 53.

5. Find the gradient for a 2D scalar field $f(\vec{r}) = e^x + \ln y$. Calculate *approximately* the value of the field $f(\vec{r})$ at point $\vec{r} = (0.05, 1.2)$ and at point $\vec{r} = (0.1, 1.1)$, using the linear approximation for the field around point $\vec{r}_0 = (0, 1)$:

   $$\delta f = f(\vec{r}) - f(\vec{r}_0) \approx \vec{\nabla} f(\vec{r}_0) \bullet \delta\vec{r}$$

   where $\delta\vec{r} = \vec{r} - \vec{r}_0$

   Naturally, the estimates you obtain should agree with the exact values of $f(\vec{r})$.

6. Find the gradient of a 3D scalar field $f(x, y, z) = z^{1/2} \exp(x + y)$. Estimate the value of the field at point $\vec{r} = (0.05, 0.1, 1.2)$ using the linear approximation for the field around point $\vec{r}_0 = (0, 0, 1)$:

   $$\delta f = f(\vec{r}) - f(\vec{r}_0) \approx \vec{\nabla} f(\vec{r}_0) \bullet \delta\vec{r}$$

   where $\delta\vec{r} = \vec{r} - \vec{r}_0$

   Compare with the exact value of $f(\vec{r})$.