Math 335-002 * Midterm Examination * March 12, 2015 * Prof. Matveev

Please show all work to receive full credit. Neither notes nor electronic devices allowed.

1. **(12pts)** Simplify the following expressions using properties of vector products. Bold letters indicate vectors, and $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are standard basis unit vectors.

   (a) $(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) \times \hat{\mathbf{k}}$ (b) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b}$ (c) $((2\hat{\mathbf{i}}) \times (\mathbf{a} \times \hat{\mathbf{i}})) \cdot \mathbf{a}$

2. **(12pts)** Sketch two x-sections (x=2 and x=3) and two y-sections (y=0 and y=1) of surface $x^2 - y^2 - z^2 - 3 = 0$. Use this information to sketch and categorize this quadratic surface.

3. **(12pts)** Find the gradient of the scalar field in $\mathbf{R}^3$, $f(\mathbf{r}) \equiv f(x, y, z) = \exp(r)$, where r is the length (norm) of the position vector $\vec{\mathbf{r}} = (x, y, z)$. Express your result only in terms of vector $\vec{\mathbf{r}}$ and its norm r.

4. **(28pts)** Consider the scalar field $F(x, y) = 2\sqrt{\ln(x) + y}$

   a) **(5pts)** Find its domain and range, and sketch the domain as a region in (x, y) plane
   b) **(5pts)** Sketch level curves F=0, F=2, F=4
   c) **(6pts)** Find the following derivatives: F_x, F_y, F_{xy} and F_{yy}
   d) **(12pts)** Find the quadratic approximation around point $\mathbf{r}_o$=(1, 1), and use it to estimate F(1.2, 1.2). Use any method you like. Note that $\mathbf{r}_o \neq 0$, therefore $\mathbf{r} = (x, y) = \mathbf{r}_o + \mathbf{h} = (1 + h_1, 1 + h_2)$

5. **(12pts)** Sketch this space-curve in $\mathbf{R}^2$, and find its length: $\mathbf{c}(t) = (t^3, 2 + t^4)$, $t \in [0, 1]$.

============= **Choose any <u>two</u> problems from the last four problems 6-9** =============

6. **(12pts)** Sketch vector field $\mathbf{u} = (2y, 1, 0)$ in the x-y plane, and find its divergence $(\nabla \cdot \mathbf{u})$ and curl $(\nabla \times \mathbf{u})$. Comment briefly on how your sketch explains the value of curl that you found.

7. **(12pts)** Find local and global extrema of function $F(x, y) = \exp(-x^2+4x-y^2)$ inside the disk $x^2 + y^2 \leq 9$

8. **(12pts)** Find the relationship between angular velocity ω, radius R, mass m and constant k for a body moving along a circular orbit due to a conservative force described by potential energy $V(\mathbf{r}) = k\exp(r)$, where r is the length of the position vector in $\mathbf{R}^2$, $\vec{\mathbf{r}} = (x, y)$ (hint: see problem **3**)

9. **(12pts)** Find the following limits, if they exist; if the limit does not exist, explain why:

   a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^2 + 2xy + y^2}$ b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y^2}{x^4 + y^4}$ c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{y}$

   In (c), use linearization or use algebra to modify and convert to a known single-dimensional limit