Math 335-002
Homework #10 * Spring 2015 * Prof. Victor Matveev

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Use cylindrical coordinates to find the mass and the center of mass of an object with density $\delta(\mathbf{r}) = x^2 + y^2$ enclosed between the z=0 plane and the paraboloid $z = 4 - x^2 - y^2$.

2. Find the line integral of the vector field $\mathbf{F}=(x^2, y^{1/3}, yz)$ along the curve given by $\mathbf{r}(t) = \left(t^2,\ e^{3t}, e^{2t}\right),\ t \in [0, 1]$.

3. Calculate the line integral of the vector field $\mathbf{F}=(y^2, -x, 0)$ over each of the following three curves connecting points A=(1,0,0) and B=(0,1,0):

a. A horizontal line connecting point A to the origin (0,0,0), followed by a vertical line connecting the origin and point B.

b. A circular arc connecting points A and B (recall that trigonometric functions parametrize this circle)

c. A straight line connecting points A and B

Compare the three results. Is **F** a conservative vector field? Calculate the curl of **F** to check your conclusion.

4. Consider a conservative force $\mathbf{F} = -\nabla f$ with potential energy f given by $f = \ln(r)$, where $r = \sqrt{x^2 + y^2}$ is the norm of position vector in $\mathbf{R}^2$. Use line integration to calculate the work done by this force over a parabolic path $y=x^2$, for x varying from 0 to 1. Compare this value with the difference in potential energy between the endpoints of the curve. Finally, find the curl of **F** to show that it is irrotational (assume F_3=0)