Math 335-002 * Spring 2015 * Homework #11

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Calculate the area of the side curved surface of the cone $x^2 + y^2 = z^2$ of height H ($z \le$ H), Hint: recall that $A(S) = \iint_S \|\mathbf{dS}\| = \iint_S \|\mathbf{T}_u \times \mathbf{T}_v\| \, du \, dv$. There are many good choices for parametrizing this surface (u and v could be Cartesian, spherical, or cylindrical variables).

2. Calculate the flux $\iint_S \mathbf{F} \cdot \mathbf{dS}$ of the vector field $\mathbf{F}=(e^x, y^2, x+y+z)$ across the surface S which is part of the coordinate plane $z=0$ lying between the curves $y=x$ and $y=x^3$ in the positive quadrant ($x\geq0$, $y\geq0$), with the normal pointing upward. Hint: no special parametrization is required, since it's a flat coordinate surface.

3. Calculate the flux (the surface integral) $\iint_S \mathbf{F} \cdot \mathbf{dS}$ of a vector field $\mathbf{F}=(y, x, \ln(x+y))$, and S is the curved side of the cylinder $x^2+y^2 = 1$ lying between the planes $z=0$ and $z=1$ in the octant $x\geq0$, $y\geq0$, $z\geq0$, with the normal pointing outward. Use variables y (or x) and z to parametrize this curved surface: $\mathbf{dS} = \left(\mathbf{T}_y \times \mathbf{T}_z\right) dy\, dz$ (Hint: the position vector will contain a square root, but everything simplifies in the end).

4. Verify the Stokes theorem $\oint_{\partial S} \mathbf{F} \cdot \mathbf{dr} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{dS}$ for the part of the curved surface $x^2 + y + z = 4$ enclosed in the first octant, and the field $\mathbf{F} = \left(y^2, 0, 0\right)$ (hint: we started this problem in class).