1. Verify the Green's Theorem for the vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = (1,\ x^2)$, and the region enclosed between the curves $y = x^2 - x$ and $y = x$. Start by sketching the region of integration.

2. Use divergence theorem to find the flux of vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = \left(\sin\left(y^2 + z\right),\ \ln\left(x^2 + z\right),\ z^3\right)$ out of the surface of the sphere $x^2 + y^2 + z^2 = 9$. Hint: this problem is extremely simple: pick the side of the Divergence Theorem that is easier to calculate.

3. Verify the Divergence Theorem for the vector field $\mathbf{F}(\vec{\mathbf{r}}) = (x,\ 0,\ z)$ and the cylindrical volume defined by $x^2 + y^2 \le 4,\ \ |z| \le 1$. Hint: the boundary of this volume is composed of three smooth surfaces.

4. **(You may submit this next week)** Verify the Divergence Theorem for the volume enclosed by the spherical cone (spherical sector) defined by $\rho \le 2,\ \phi \le \dfrac{\pi}{3}$, for the vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}}) = \left(x,\ 0,\ z^2\right)$. Note that this closed surface consists of two different surfaces: the top spherical surface and the side conical surface.

5. Show that the volume of a three-dimensional object can be calculated by double integration over its surface instead of triple integration, by plugging a simple vector field of form $\mathbf{F}(\vec{\mathbf{r}}) = (ax,\ by,\ cz)$ into the divergence theorem (*a*, *b*, and *c* are constants), and finding conditions on constants *a*, *b*, and *c* so that the flux integral equals the volume if the object. Use this method to calculate the volume of a sphere of radius R.

Integral Theorem summary:

• Stokes Theorem: $\displaystyle\oint_{\partial S} \mathbf{F}\cdot d\mathbf{r} = \iint_S \nabla\times\mathbf{F}\cdot d\mathbf{S}$

$\Rightarrow$ Green's Theorem: $\displaystyle\oint_{\partial D} \left(F_1\, dx + F_2\, dy\right) = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\, dy$

• Divergence Theorem: $\displaystyle\oiint_{\partial W} \mathbf{F}\cdot d\mathbf{S} = \iiint_W \nabla\cdot\mathbf{F}\ dV$