Math 335-002 * Homework #4 * Due date: February 19

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Find the domain and range of scalar field in $\mathbf{R}^2$, $f(x, y) = \ln(x - y^2)$, and sketch its level curves. To do this, solve the equation $f(x,y)=k$=const, and plot these curves for several values of k

2. Find the gradient of a scalar field in $\mathbf{R}^2$, $f(\mathbf{r}) = \mathbf{r} \cdot \mathbf{a}$, where $\mathbf{a}$ is a constant vector. For $\mathbf{a}$= (1, 1), sketch separately the scalar field (by showing its level curves) and its gradient. Keep in mind that the gradient is a vector field.

3. Find the gradient for a 3D scalar field in $\mathbf{R}^3$, $f(\mathbf{r}) = z\, e^x (1 + \ln y)$ (it will have 3 components, since there are now 3 coordinates). Calculate *approximately* the value of the field $f(\mathbf{r})$ at point $\mathbf{r}$ = (0.05, 1.1, 1.2), using the linear approximation for the field around point $\mathbf{r}_o$= (0, 1, 1), and compare with exact value of this function

   Reminder of linear approximation: $f(\mathbf{r}) \approx f(\mathbf{r_o}) + \nabla f(\mathbf{r_o}) \cdot (\mathbf{r} - \mathbf{r_o})$

4. Calculate the gradient of scalar field in $\mathbf{R}^3$, $f(x, y, z) = \ln(\rho)$, where ρ is the distance from the origin (the spherical variable ρ), and express the result only in terms of position vector $\mathbf{r}$ and its length, which is equal to ρ

   Hint: recall the example we did in class, where we found the gradient of function $r = \sqrt{x^2 + y^2}$ in $\mathbf{R}^2$ to equal $\nabla r = \nabla\sqrt{x^2 + y^2} = \frac{\mathbf{r}}{\|\mathbf{r}\|} = \frac{\mathbf{r}}{r} = \hat{\mathbf{r}}$, where r is the polar variable, equal to distance from origin in in $\mathbf{R}^2$, and $\mathbf{r}$ is the position vector.