Math 335-002 * Homework #5 * Due date: February 26

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Sketch this space-curve in $\mathbf{R}^2$: $\mathbf{c}(t) = (\, 2\cos(t/2), \sin(t/2)\,)$, $t \in \mathbf{R}$; calculate and plot its tangent vector at points $t=\pi$ and $t=2\pi$.

2. Sketch this space-curve in $\mathbf{R}^2$: $\mathbf{c}(t) = (\, t^3\,, t^2\,)$, $t \in [-2, 2]$. Is this space-curve smooth for all values of t ?

3. Calculate the quadratic approximation to function $f(\mathbf{r}) = \ln\left(e^{x+y} + x + z\right)$, and use it to estimate $f(0.02, 0.02, 0.02)$. Verify your answer by comparing with exact value of this function.

Quadratic approximation is given by:

$$f(\mathbf{r}_o + \mathbf{h}) \approx f(\mathbf{r}_o) + Df(\mathbf{r}_o)\cdot\mathbf{h} + \frac{1}{2}\mathbf{h}^T D^2 f(\mathbf{r}_o)\mathbf{h}$$

where $Df(\mathbf{r}_o) \equiv \nabla f(\mathbf{r}_o)$ and $\mathbf{h}^T D^2 f(\mathbf{r}_o)\mathbf{h} \equiv (h_1,\, h_2,\, h_3) \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix}_{\mathbf{r}_o} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$