Math 335-002 * Homework #6 * Due date: March 5, 2015

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Section 3.1:
   A function satisfying the Laplace's equation is called *harmonic*. Which of the following functions is/are harmonic?

   (*a*) $f(x,y) = \cos(2x)\,\sinh(2y)$

   (b) $f(x,y) = \cos(3x)\,\exp(-2y)$

   (c) $f(x,y) = \log\left(x^2 + y^2\right)$

2. Problem 3.1.12(b), page 157: show that $T(x,y,t) = e^{-kt}\left(\cos x + \cos y\right)$ satisfies the two-dimensional heat equation:

$$\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

3. Problem 3.3.6, page 182: Find and classify the critical points (i.e. determine whether they are local maxima, minima, or saddles):

$$f\left(x,y\right) = 3x^2 + 2xy + 2x + y^2 + y + 4$$

4. Problem 3.3.44, page 184: Find absolute (global) extreme points of function $f(x,y) = 1 + xy + x - 2y$ on a triangular region with vertices (1, −2), (5, −2) and (1, 2)

   Hint: one of the three boundaries will require a parameterization using equation of line (you can use equation of line in any form you like).

5. Find the coefficient *k* in the following "relativistic" correction to the familiar expression for the kinetic energy of a body of resting mass m_o moving with speed *v*, by Taylor-expanding the following relativistic expression to *second* order in variable $x=(v/c)^2$:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \approx \underbrace{m_o c^2}_{\text{Rest Energy}} + \underbrace{\frac{m_o v^2}{2} + k\,v^4}_{\text{Kinetic Energy}}$$

   Hint: I did this in class, you only have to find the coefficient *k* of the correction term using the next Taylor term in the expansion of $(1 - x)^{-1/2}$