Math 335-002 * Homework #7 * Due Wednesday March 11, 2015

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Section 4.1: Use vector calculus to show that an object moving at constant speed has velocity that is perpendicular to the force acting on this object.

   Hint: we solved an almost identical problem in class: differentiate $\|\mathbf{v}\|^2$ and apply differentiation product rule to immediately obtain the desired result

2. Section 4.1:
   A particle is moving along a circle in the plane $\mathbf{r}(t) = (R\cos\omega t,\ R\sin\omega t)$ under the influence of force $\mathbf{F}(\mathbf{r}) = -\nabla V(\mathbf{r})$, where $V(\mathbf{r}) = k \ln r$ is the potential energy. Here k is a constant, and $r \equiv \|\mathbf{r}\| = \sqrt{x^2 + y^2}$. Find the relationship (equation) connecting constant k, the radius of orbit R, and angular velocity ω

   Hint: apply the second Newton's law: $m\mathbf{a}(t) = \mathbf{F}(\mathbf{r}(t)) \Rightarrow m\dfrac{d^2\mathbf{r}}{dt^2} = -\nabla V(\mathbf{r}(t))$;
   equating the two sides of this equation immediately gives the result. Review the last problem of homework #4 for the gradient calculation.

3. Section 4.2:
   Find length or arc (curve) given by $\mathbf{c}(t) = (t^2\sin t,\ t^2\cos t,\ t^3)$ on the interval $t \in [0, \pi]$. Hint: arc-length is given by $\int_a^b \left\|\dfrac{d\mathbf{c}}{dt}\right\| dt$

4. Sections 4.3-4.4:
   Find the divergence and curl of the vector field $\mathbf{F}(\mathbf{r}) = (y^2,\ x^2,\ 0)$, and sketch this field in the x-y plane (since F_3=0, it lies entirely in the x-y plane).

5. Sections 4.3-4.4: Calculate the divergence and the curl of the vector field
   $$\mathbf{F}(\mathbf{r}) = (y\cos x,\ \sin(z\,x),\ \ln(y\sin z))$$

$$div\ \mathbf{F} \equiv \nabla\cdot\mathbf{F} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(F_1, F_2, F_3) \equiv \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$curl\ \mathbf{F} \equiv \nabla\times\mathbf{F} \equiv \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z},\ \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x},\ \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$