Math 335-002 * Homework #8 * Due Monday March 30, 2015

Please show all work in detail to receive full credit. Late homework is not accepted.

1. Let **F** and **G** denote two differentiable vector fields in $\mathbf{R}^3$. Prove the following product rule by calculating the left-hand side and the right-hand side of this equation in term of components of these fields, F_k and G_k (where k=1,2,3)

$$\operatorname{div}(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\operatorname{curl}\mathbf{F} - \mathbf{F}\cdot\operatorname{curl}\mathbf{G}$$

2. Suppose f is a C^2 (twice differentiable) scalar field in $\mathbf{R}^3$. Which of the following expressions are meaningful, and which are nonsence? For those which are meaningful, decided whether the expression defines a scalar field or a vector field:

a) curl (grad f)
b) grad (curl f)
c) div (grad f)
d) grad (div f)
e) curl (div f)
f) div (curl f)
g) grad (grad f)

3. Sketch the region of integration, change the order of integration, and evaluate:

$$\int_0^3 \int_{y^2}^9 y\cos\left(\frac{\pi x^2}{2}\right) dx\, dy$$

4. Consider the integral $\iint_D \frac{y^3\, dx\, dy}{\sqrt{x^2+y^2}}$, where integration region D is determined by the conditions $\frac{1}{2} \le y \le 1,\ x^2+y^2 \le 1,$ and $x \ge 0$.

a) Sketch the region of integration

b) Set up limits of integration for two different integration orders

c) Calculate this integral using integration order ($dy\, dx$).