Math 340 * Exam 2 * April 5, 2012 * Victor Matveev

Please show all work for each problem. Simplify all answers as much as possible.

All electronic devices must be turned off. Notes are not permitted.

1) (**22pts)** Compute the following approximations for $\int_{-1}^{1} x^4 dx$:

a) Find the Simpson's approximations S_2 and S_4 using n=2 and n=4 subintervals, respectively. Compare each result with the exact value of the integral, up to three decimal digits.

b) Correct the value of $\boldsymbol{S_2}$ (not S_4) using its asymptotic error, $-\frac{h^4}{180}\left[f^{(m)}(b) - f^{(m)}(a) \right]$, where $f^{(m)}(x)$ is an appropriate derivative of $f(x)$. Simplify as much as possible.

c) Find the Richardson's extrapolation using values S_2 and S_4 obtained in part (a). Simplify as much as possible.

2) **(22pts)** Find values w_1, w_2 and x_2 so that the following integration rule has degree of precision of 2 by applying this rule to $f(x)$ =1, x, and x^2, and use the resulting integration rule to estimate $\int_0^{5\pi/9} \frac{\cos(x)}{\sqrt{x}} dx$. Why shouldn't you use the Simpson's or Gaussian integration rules to estimate this integral?

$$\int_0^h \frac{f(x)}{\sqrt{x}} dx = w_1 f(0) + w_2 f(x_2)$$

3) **(22pts)** What does the following expression approximate for small values of h? To answer this question, expand the first and the third terms in the numerator in a Taylor series about x_o, up to third order in h . You may leave out the remainder term. What is the error of this finite difference? To check your answer, apply this formula to $f(x)=x^2$

$$Df(x_0) = \frac{f(x_0 - 3h) - 4f(x_0) + 3f(x_0 + h)}{6h^2}$$

4) (**22pts)** Consider the function $f(x) = \frac{1}{1+x}$ on the interval $[-0.5,\ 0.5]$:

a) Use Newton's divided differences to find the quadratic interpolating polynomial approximation for $f(x)$ using its values at $x_0 = -0.5,\ x_1 = 0$ and $x_2 = 0.5.$ Make a rough sketch of this interpolating polynomial

b) Compare this interpolating polynomial with the quadratic Taylor polynomial for $f(x)$ about x_1=0. Without doing any calculations, explain which of these two quadratic polynomials is a more accurate approximation to $f(x)$ at x=0.47.

You may choose between problems 5 and 5':

5) (**12pts)** Find and make a rough sketch of the **piece-wise quadratic spline** $q(x)$ on the interval [0, 2] that satisfies conditions listed below (note that $q(x)$ is formed by two quadratic polynomials with domains [0, 1] and [1,2], respectively, and recall that a quadratic spline has a continuous first derivative):

$$q(0) = 1,\ q(1) = 0,\ q(2) = 2,\ \frac{dq}{dx}(2) = -1$$

5') (**12pts)** Derive the formula for Richardson's extrapolation of Simpson's integration rule using values S_{2n} and S_{3n} instead of S_n and S_{2n}