Math 340 * Exam 1
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Please show all work for each problem. Neither calculators nor notes are permitted.

1) (**22pts)** For what range(s) of x values does each function below result in *complete* loss of significance when evaluated in double precision? What result would MATLAB produce for those values of x? Using Taylor polynomials, modify each expression to avoid loss of significance for these problematic values of x. Keep only *two* dominant (largest) terms in your modified expression in each case.

a) $\dfrac{\exp(2x^2)-(1+6x^2)^{1/3}}{x^4}$ **b)** $\dfrac{x^2}{\sqrt{x^2+1}-\sqrt[3]{x^3+1}}$

2) (**22pts)** Find a rational approximation to $\sqrt[3]{5}$ using two different methods, and find an upper bound for the error of each approximation:

a) Apply *one* iteration of the Newton's method to an appropriately chosen function, starting with initial guess x_o=2. Find the bound on error $\varepsilon_1 = \dfrac{f''(c_0)}{2f'(x_0)}\varepsilon_0^2$, approximating the initial error as $\varepsilon_0 = \alpha - x_0 \approx x_1 - x_0$.

b) Noting that $\sqrt[3]{5} = \sqrt[3]{8-3} = 2\sqrt[3]{1-3/8}$, use the linearization of the function $\sqrt[3]{1-x}$ to estimate $\sqrt[3]{5}$, and use the Taylor remainder formula to find the upper bound for the error of this linear approximation.

3) (**22pts)** For each of the following iterations, find the fixed point(s) or find an interval containing the fixed point(s). Indicate the fixed points graphically, and categorize their stability. For each stable fixed point, find the convergence order and convergence type (i.e. indicate whether it is monotonic or non-monotonic) :

a) $x_{n+1} = 1 - \dfrac{x_n^3}{3}$ **b)** $x_{n+1} = \dfrac{2}{3}x_n + \dfrac{1}{3x_n^2}$

4) **(18pts)** In this problem you will derive and examine the secant method iteration:

a) Write down the equation of line connecting any two points on the graph of function $f(x)$ corresponding to argument values x_o and x_1

b) Find the intercept of this line with the horizontal axis, and use this result to give an expression for x_{n+1} as a function of x_n and x_{n-1}

c) Does the secant method converge for any function and for any initial points x_o and x_1? If yes, explain why; if no, use a rough sketch to illustrate a diverging sequence resulting from repeated application of the secant iteration.

5) **(16pts)** Consider the following MATLAB program. Figure out the value of F in terms of x after each iteration of the loop (note that the loop variable runs "backwards", from k=8 down to k=2, with a step of –2). What is the final value of F, as a function of x? What elementary function does this program compute?

```
F = function myFun(x)
x2 = x*x;
F = 1;
for  k = 8 :  – 2 : 2
   F = 1  –  x2 * F / (k * (k-1)) ;
end
```