Fall 2015 * Math 430 * Math 635 * Prof. Victor Matveev
Homework 2
Please explain all steps in your work briefly

1. Calculate the Nernst potential (also known as the reversal potential or the equilibrium potential) for Na^+ at room temperature (20°C), assuming that $[Na^+]_{in}$=15mM and $[Na^+]_{out}$=150mM. How much different will the reversal potential be at body temperature (37°C)? Compare your results with the approximate formula V_X = 62mV $\log_{10}$ $([X]_{out}/[X]_{in})$

   Recall that $V_{Na} = \frac{RT}{zF}\ln\frac{[\mathrm{Na}^+]_{out}}{[\mathrm{Na}^+]_{in}} = \frac{k_B T}{q}\ln\frac{[\mathrm{Na}^+]_{out}}{[\mathrm{Na}^+]_{in}}$, where *q* is the charge of a single Na^+ ion

   Note: you can use the web to look up values of fundamental constants appearing in this equation.

2. Consider the "current-clamped" passive membrane with R=100 MΩ, C=100 pF, I = 0.2 μA, and equilibrium ("rest") potential of V_R=-60 mV. As we showed in class, this equation can be written as (carefully retrace our steps if in doubt):

$$\begin{cases} \tau_m V' = -(V - V_R) + RI \\ V(0) = V_R \end{cases}$$

   a) Calculate the membrane time constant τ_m

   b) Write down and plot the solution, V(t) (shift the variable to get rid of all constants before solving)

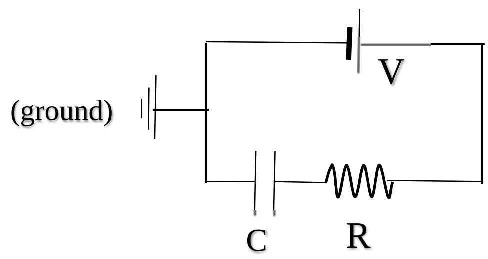
   c) Calculate the amount of time it takes for the potential to reach –50mV

3. Solve two of the following differential equations using separation of variables (see an example on the next page). Make sure to solve your integration results for Y(t), and plot the solution as a function of *t*.

$$\begin{cases} \frac{dY}{dt} = t^2 Y \\ Y(0) = 1 \end{cases} \qquad (b)\ \begin{cases} \frac{dY}{dt} = Y^2 \\ Y(0) = 1 \end{cases}$$

4. Modify the program in problem 3 of homework 1 so that it computes a cosine function of x, instead of exponential of x. Hint: recall that $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$. Note that there are several ways to do it; think of the most efficient (simplest) way to implement this calculation.

5. Consider a resistor R and capacitor C connected in a series, with a voltage difference of V applied across these two elements at time *t*=0 (**see Figure on next page**). Find the current flowing around this circuit, as a function of time *t*. Hint: write down the three equations for three unknowns (one of the three equations is $V=V_R+V_C$). Solve for $V_C(t)$ using integration, and then calculate the current, $I(t)$



Circuit for problem 5

Separation of variables example: let's solve $\begin{cases} \dfrac{dY}{dt} = \dfrac{e^{-Y}}{t+1} \\ Y(0) = 1 \end{cases}$

1. Separate the variables: $e^{Y}dY = \dfrac{dt}{t+1}$

2. Integrate both sides $\Rightarrow$ $\left[e^{Y}\right]_{Y(0)}^{Y(t)} = \left[\ln(\tau+1)\right]_0^t \Rightarrow e^{Y(t)} - \underbrace{e^{Y(0)}}_{e} = \ln(t+1) - \underbrace{\ln 1}_{0}$

3. Solve for Y(*t*): $e^{Y(t)} = \ln(t+1) + e \;\; \Rightarrow$ $\boxed{Y(t) = \ln\left(e + \ln(t+1)\right)}$