Fall 2015 * Math 430 * Math 635 * Prof. Victor Matveev
Homework 3 * Due date: September 25

1. Consider a passive cell which at time zero starts receiving an exponentially decreasing current $I(t) = I_o e^{-\beta t}$

$$\begin{cases} \tau_m V' = -(V - V_R) + R I_o e^{-\beta t} \\ V(0) = V_R \end{cases}$$

Calculate the potential of the cell as a function of time, using the method of *constant variation*:

- Shift the potential by introducing variable $v(t) = V(t) - V_R$
- Solve the linear homogeneous equation $\begin{cases} \tau_m v' = -v \\ v(0) = C \end{cases}$
- Replace constant C with time-dependent C(*t*), then plug it into equation $\tau_m v' = -v + R I_o e^{-\beta t}$ to find and solve the equation for C(*t*) (note that C(0)=v(0)=0)
- Write down the solution V(t), and make a rough plot

2. Using linear stability analysis, find all equilibria, categorize their stability, and make a rough sketch of the solution of these nonlinear autonomous differential equations, for the given initial condition:

$$(a)\ \begin{cases} Y' = \sin Y \\ Y(0) = 0.2 \end{cases} \qquad (b)\ \begin{cases} Y' = \sin Y \\ Y(0) = -0.2 \end{cases} \qquad (c)\ \begin{cases} Y' = Y(1-Y) \\ Y(0) = 0.1 \end{cases}$$

3. Check your solution to problem 2(c) using separation of variables, noting that $\frac{1}{Y(1-Y)} = \frac{1}{1-Y} + \frac{1}{Y}$

4. Find the equilibrium of the following autonomous problem, and show that linear stability analysis is insufficient to categorize its stability. Then, make a graph of dY/dt as a function of Y (the "phase plot") to figure out the stability of the equilibrium, and make a rough sketch of the solution, Y(t)

$$\begin{cases} Y' = -Y^3 \\ Y(0) = 1 \end{cases}$$

5. Write a short MATLAB program that finds the sum of all Fibonacci numbers that are less than L=1000. Recall that Fibonacci numbers are an integer sequence in which each number is a sum of two preceding numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, … The basic structure of the program appears on the next page.

```
function S = FibonacciSum(L)
S = … ;
….
while num < L
      S = S + num;
      num = …
      ….
end
```