Fall 2015 * Math 430 * Math 635 * Prof. Victor Matveev
Homework 4 * Due date: September 30

1. Modify the posted Euler Method program very slightly so that it accepts the time step *h* as its argument, instead of number of partitions *N*. The header of the modified myEuler.m function will be

```
function [T, Y] = myEuler(f, Y0, tTotal, h)
```

Make sure the program still produces the same output as the original program for appropriate ***h*** values

Extra credit will be given if you also successfully remove the T(n) calculation from the for-loop (which is inefficient and suffers from double precision round-off error accumulation with each iterations).

2. Use the forward Euler method program to numerically solve the following equation on the interval $0 < t < 5$, using several different values of time step *h*: *h*=0.1, *h*=0.5 and *h*=1. Show all three numerical solutions on the same Matlab plot, as well as the exact solution curve you found in homework 3 (make sure to label your plots nicely). **Finally, compute the first three iterations (Y_1, Y_2, Y_3) manually for *h*=0.5, to check your program.**

$$\begin{cases} Y' = Y(1-Y) \\ Y(0) = 0.2 \end{cases}$$

3. Consider the "current-clamped" passive cell with R=50 MΩ, C=300 pF, I=0.3 nA, and equilibrium ("rest") potential of V_R=-60 mV. As we know, the relevant equation is (see problem 1 of hw #2):

$$\begin{cases} V' = -\dfrac{V - V_{RI}}{\tau_m} \quad \text{where } V_{RI} \equiv V_R + R\,I \\ V(0) = V_R \quad \text{(cell is at rest initially, when the currrent pulse arrives)} \end{cases}$$

a) Find the constants V_{RI} and τ_m, expressing their values in units of mV and ms, respectively.

b) Use the Euler's method to solve this equation numerically on the interval 0 < t < 100 ms with step *h*=1 ms. Plot this solution along with the exact analytic solution. Assume all values are in units of mV and ms throughout the program.

c) Review problem 1 of homework #2 to calculate the amount of time it takes for the potential to reach –50mV, and use the plot in part "b" to verify your answer (add a horizontal line at V=-50 to your Matlab *V*(*t*) plot)

4. Modify the Euler program slightly so that is can solve a non-autonomous problem, $Y'=f(t, Y)$, and solve the following differential equation with time steps *h*=0.1, *h*=0.2 and *h*=0.5 on the interval $0<t<5$. **Compute the first three iterations for *h*=0.5 manually to check your program**

$$\begin{cases} Y' = Y - t \\ Y(0) = 0.5 \end{cases}$$

Hint: f = @(t, Y) Y – t; there is only one more tiny change in the whole program.

5. Use the program you made for problem 4 to numerically solve problem 1 of hw #3 on the interval 0 < t < 50 ms, with time step *h* =1 ms. Plot this numerical solution along with the exact analytic solution (see hw #3) using Matlab.

$$\begin{cases} \tau_m V' = -\left(V - V_R\right) + R\,I_o e^{-\beta t} \\ V(0) = V_R \end{cases}$$

Use units of mV for potential, and ms for time, and assume the following parameter values:

$$\tau_m = 5ms,\ \beta = 0.1ms^{-1},\ R = 10M\Omega,\ I_o = 5nA,\ V_R = -60mV$$