Fall 2015 * Math 430 * Math 635 * Prof. Victor Matveev
Homework 5 * Due date: October 8

1. Consider the following differential equation, with a parameter *I* : $\frac{dV}{dt} = f\left(V;I\right) = V^3 + IV$

a) Make a “phase portrait” of dynamics for the following values of model parameter: $I = -1$ and $I = 1$

b) Find all equilibria V^* as a function of parameter *I*; note whether they exist for all values of *I* or not.

c) Find critical (bifurcation) value I_{cr} for which some equilibria “collide”. Compare your result with the critical value obtained from the non-hyperbolicity condition $\lambda = \frac{df}{dV}\left(V^*, I_{cr}\right) = 0$ (where V* satisfy $f\left(V^*; I_{cr}\right) = 0$)

d) Examine the stability of all equilibria V^*, and sketch the bifurcation diagram, V^* vs *I* , indicating stable equilibria with a solid curve, and unstable equilibria with a dashed curve.

e) Check the non-degeneracy and transversality conditions (see p. 74) to determine whether the bifurcation is of saddle-node type.

2. Consider the parameter-dependent model $\frac{dV}{dt} = f\left(V;I\right) = V^3 - V + I$

a) Use Matlab to plot $f\left(V;I\right)$ for several values of *I* from –1 to +1 on the same graph (use “hold on”), together with the d*V*/dt = 0 horizontal line. Use this plot to estimate the bifurcation values I_{cr} (there are two bifurcation).

b) Find the bifurcation points from the equilibrium condition $f\left(V^*; I_{cr}\right) = 0$ and the non-hyperbolicity condition $\lambda = \frac{df}{dV}\left(V^*, I_{cr}\right) = 0$, and compare with your estimate in part “a”.

c) Make a rough plot of the bifurcation diagram, V^* vs *I*, indicating stable and unstable equilibria with solid and dashed curves, respectively.

d) If we use this differential equation in a model of excitable cell of integrate-and-fire type (with a spike and reset inserted “by hand”), what would be the threshold value of potential V_{th} above which a spike is always produced?

3. Run the posted programs **IandF.m** and **IandFquadratic.m** to see a movie of the two Integrate-and-Fire models learned in class. Use the output plot produced by these two programs to estimate the inter-spike-interval in each case (Note that the time scale is different in the two programs), and compare with the values you obtain from the formulas for the inter-spike intervals derived in class. All parameters values are spelled out within the code.

4. For each program (**IandF.m** and **IandFquadratic.m**), find the critical value of current pulse amplitude $\mathbf{I_{amp}}$ for which no spike is obtained within the given window; give the answer with precision of 3 decimal digits.

5. Run **IandF.m** code with two lines replaced with the following statements; explain in one sentence what you see. Then, once again find the minimal value of $\mathbf{I_{amp}}$ (to 3 decimal digits of precision) below which no spikes are produced:

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Iamp = 15;
Current = @(t) Iamp * (t > 100) .* exp(-(t-100)/200);
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6. Calculate the spike period of the quadratic integrate-and-fire model in which the spike peak and the reset potential are unbounded (note that V(t) “blows up” from –∞ to +∞ in finite time). Hint: arctan(–∞)=–π/2, arctan(+∞)=π/2

$$\begin{cases} \frac{dV}{dt} = V^2 - b + I \\ V\left(t_k^-\right) = V_{spike} \to +\infty \;\Rightarrow\; V\left(t_k^+\right) = V_{reset} = -\infty \end{cases}$$