Fall 2015 * Math 430 * Math 635 * Prof. Victor Matveev
Homework 6 * Due date: October 15

1. Consider the linear problem $\begin{cases} x' = -x + y \\ y' = 2y \\ x(0) = 2; \;\; y(0) = 0.1 \end{cases}$

a) Sketch the nullclines, and indicate the flow along the nullclines and along coordinates axes, by drawing arrows

b) Sketch the vector field in the rest of the 2D phase plane

c) Use your vector field sketch to make a rough sketch of the solution, x(t) and y(t), for the given initial conditions

d) Find eigenvalues and eigenvectors, and use the eigenvalues to classify the equilibrium type

e) Find the exact solution using: $\mathbf{Y}(t) = c_1 e^{\lambda_1 t}\, \mathbf{v}_1 + c_2 e^{\lambda_2 t}\, \mathbf{v}_2$ (you have to determine c_1 and c_2 from the initial condition). Make sure to check that your solution satisfies the equations and the initial conditions

f) Check your conclusions by modifying the posted program “PhasePlane2D.m” (make appropriate changes in equations, initial conditions, eigenvectors/eigenvalues, and adjust the plot coordinate ranges appearing in the “axis” statements)

2. Consider the non-linear system $\begin{cases} x' = \sin(x - y) \\ y' = \sin(x) \end{cases}$. Since $\sin u \approx u$ for small values of the argument *u*, this nonlinear system can be approximated by $\begin{cases} x' \approx x - y \\ y' \approx x \end{cases}$ sufficiently close to the zero equilibrium $x^*=y^*=0$

a) Find the eigenvalues of the linearized system, and characterize the stability of the equilibrium. Hint: in this case eigenvalues form an imaginary pair $\lambda = a \pm i\, b$; Stability will depend on the value of a.

b) Compare the numerical solution of the linear system and the numerical solution of the non-linear system, using two modifications of the supplied program “PhasePlane2D.m” (modify the program to change the equations, the initial conditions, and the plot coordinate ranges in the “axis” commands; remove the final part of the program where the exact solution is calculated)**.** Use obtained numerical results to verify your conclusion about the stability of the zero equilibrium.

3. Consider the non-linear problem $\begin{cases} x' = -x + ay + x^2 y \\ y' = b - ay - x^2 y \\ x(0) = 1; \;\; y(0) = 1 \end{cases}$ where a=0.08 and b=0.6 are constant parameters.

a) Find the equilibrium, and analyze its linear stability by examining the eigenvalues of the Jacobian at the equilibrium point (hint: sum the two equations when solving for the equilibrium)

b) Modify the posted program “PhasePlane2D.m” to observe and describe the dynamics of the trajectory when x(0)=y(0)=1, and verify your conclusions about the stability of the equilibria. Make sure to integrate over a long time interval.