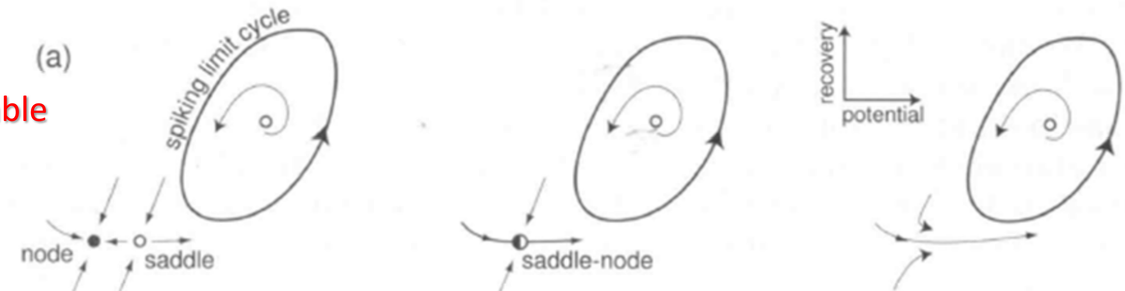
Fall 2015 * Math 430 * Math 635 * Prof. Victor Matveev

Lecture 17: Excitability Class and Bifurcations
Figures from E.M. Izhikevich “Dynamical Systems in Neuroscience”

Four simplest (co-dimension 1) bifurcations from rest to periodic spiking (page 12)

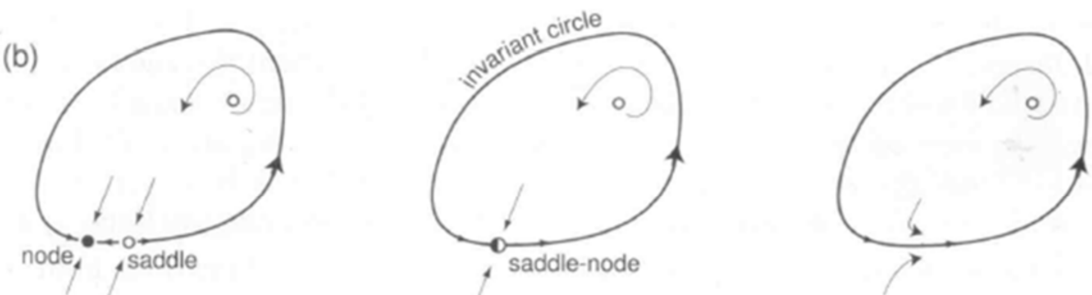
Bistable



Class 2 excitability

← Homoclinic Orbit bifurcation

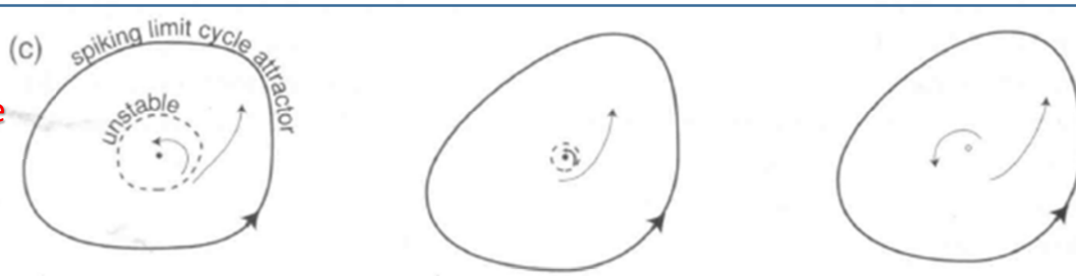
(a) Saddle-node near saddle homoclinic orbit bifurcation



Class 1 excitability

(b) Saddle-node on invariant cycle (SNIC)

Bistable



Class 2 excitability

← Fold bifurcation

(c) Subcritical Andronov-Hopf near fold bifurcation

Class 2 excitability

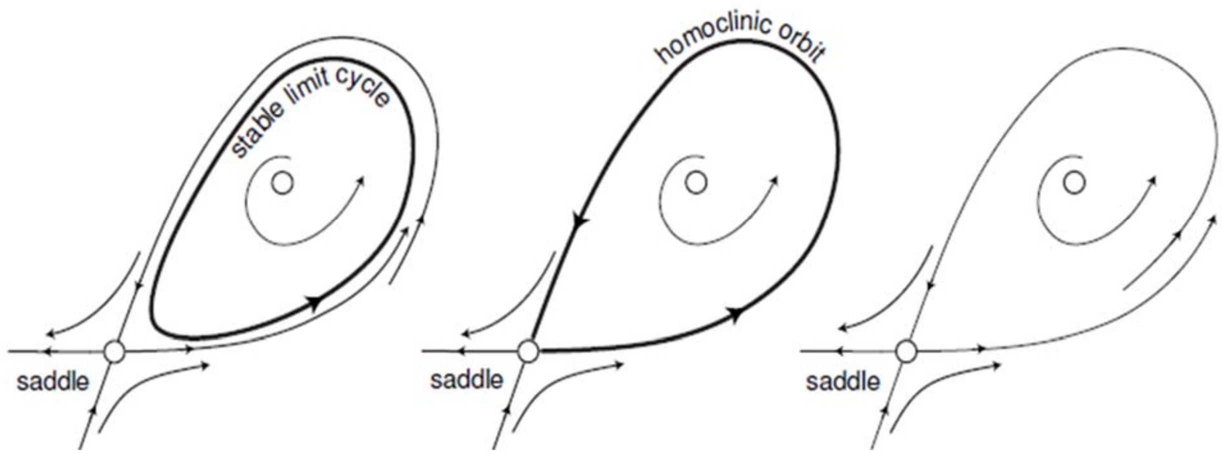
(d) Supercritical Andronov-Holpf

Figure 1.12: Four generic (codimension-1) bifurcations of an equilibrium state leading to the transition from resting to periodic spiking behavior in neurons.

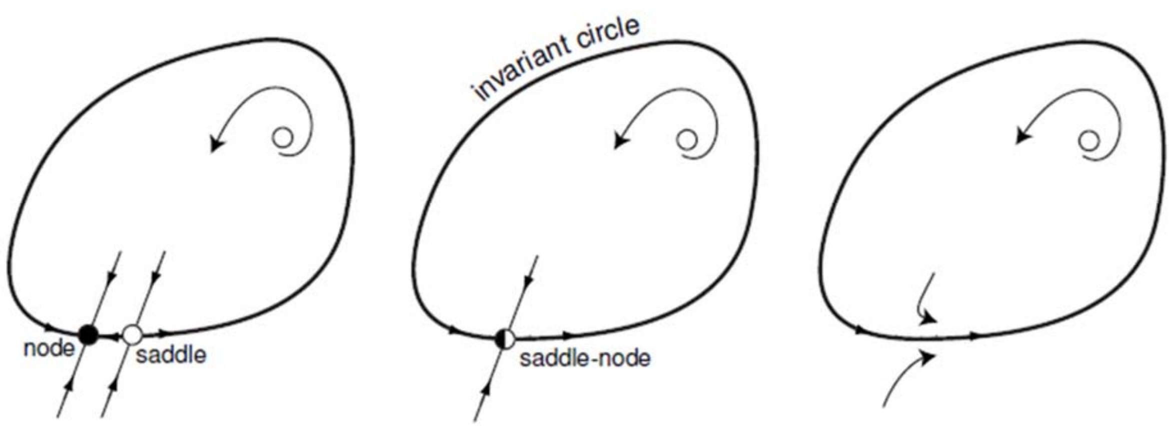
Bifurcations involving a Homoclinic Orbit

Difference between the SHOB and SNIC as the parameter crosses the critical value:

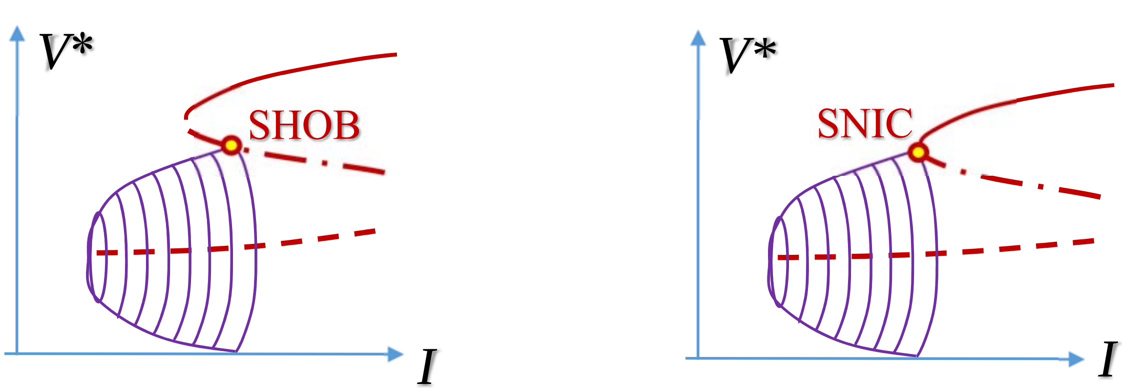
Saddle Homoclinic Orbit Bifurcation ("SHOB"):



Saddle-Node on Invariant Cycle/Circle ("SNIC"):



…Dynamics are quite different (even the type of fixed point at the critical point is different), despite similar-looking bifurcation diagrams:



Bistability between rest and spiking

In cases (a) and (c) (see page 1) there is bistability between rest and periodic spiking:

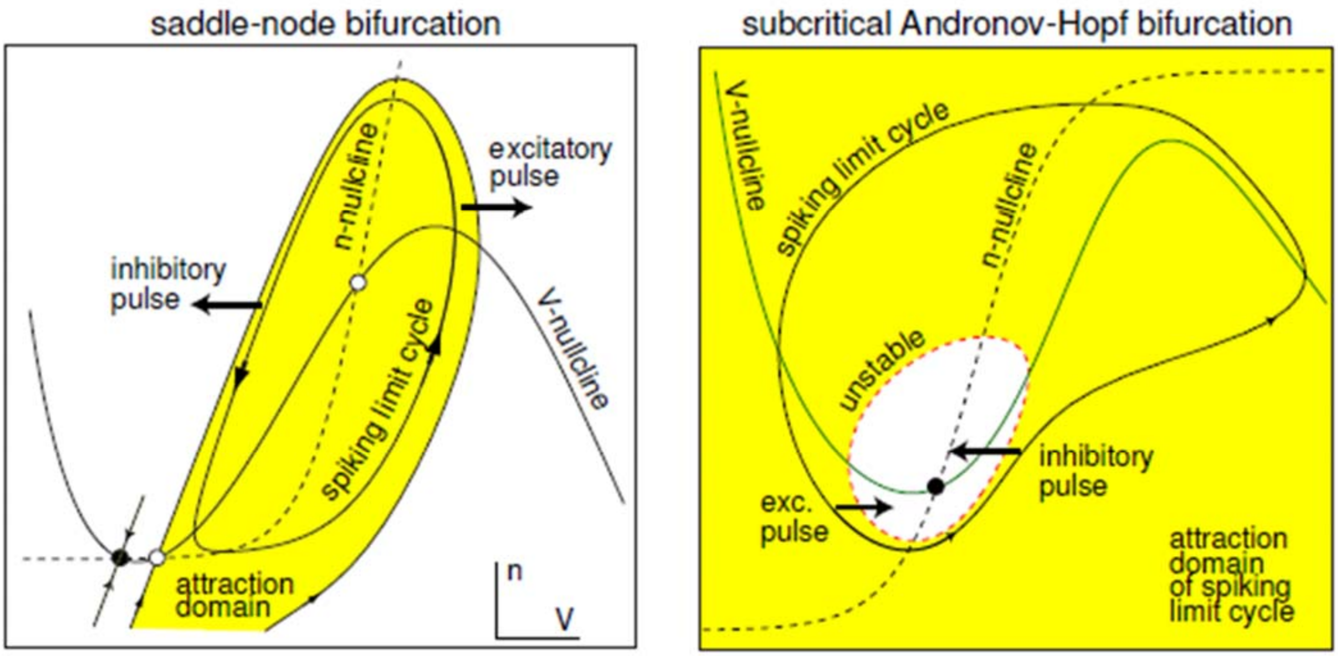
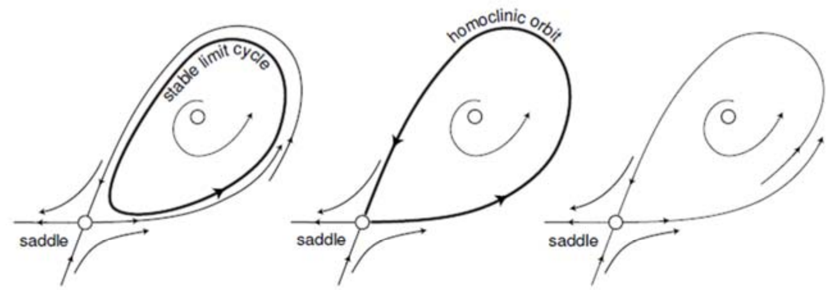


FIGURE 7.9. Coexistence of stable equilibrium and spiking limit cycle attractor in the $I_{\mathrm{Na,p}}+I_{\mathrm{K}}$-model. *Left:* The rest state is about to disappear via saddle-node bifurcation. *Right:* The rest state is about to lose stability via subcritical Andronov-Hopf bifurcation. Right (left) arrows denote the location and the direction of excitatory (inhibitory) pulse that switches spiking behavior to resting.

The arrows show the effect of excitatory or inhibitory perturbations (“kicks”) at different phases of the spike in these two distinct cases.

Homoclinic Orbit Bifurcation in detail



a. supercritical saddle homoclinic orbit bifurcation

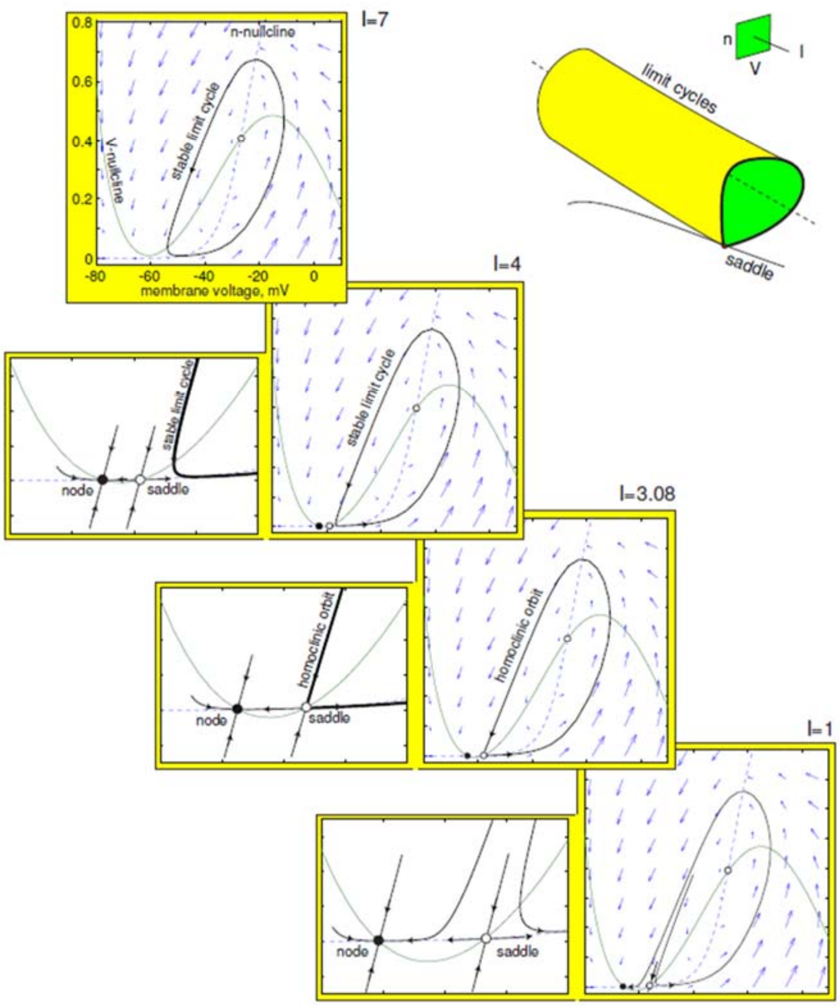


FIGURE 6.28. Saddle homoclinic orbit bifurcation in the $I_{\mathrm{Na,p}}+I_{\mathrm{K}}$-model with parameters as in Fig. 4.1a and fast K^+ current ($\tau(V) = 0.16$). As the bifurcation parameter I decreases, the stable limit cycle becomes a homoclinic orbit to a saddle.

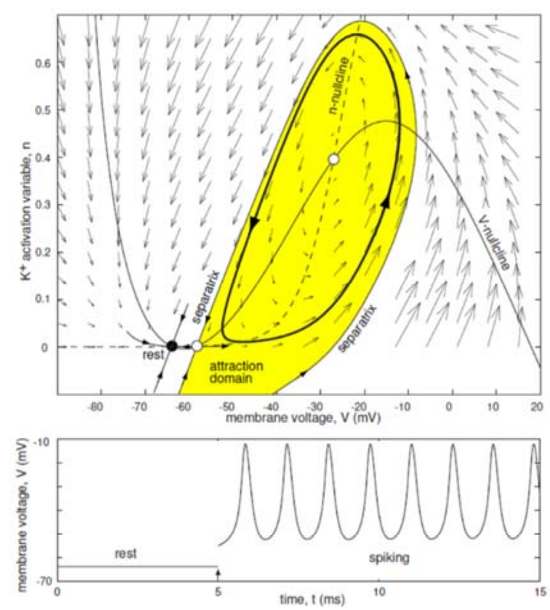


FIGURE 4.26. Bistability of rest and spiking states in the $I_{\mathrm{Na,p}}+I_{\mathrm{K}}$-model (4.1, 4.2) with high-threshold fast ($\tau(V) = 0.152$) K^+ current and $I = 3$. A brief strong pulse of current (arrow) brings the state vector of the system into the attraction domain of the stable periodic orbit.

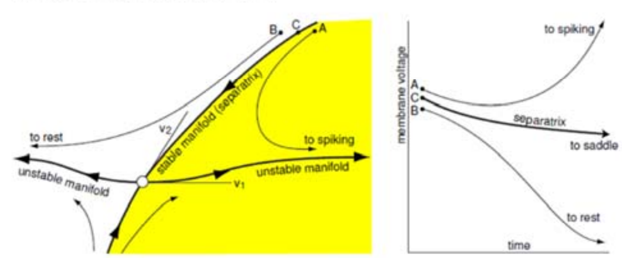


FIGURE 4.27. Stable and unstable manifolds to a saddle. Eigenvectors v_1 and v_2 correspond to positive and negative eigenvalues, respectively.

Fold of limit cycles bifurcation

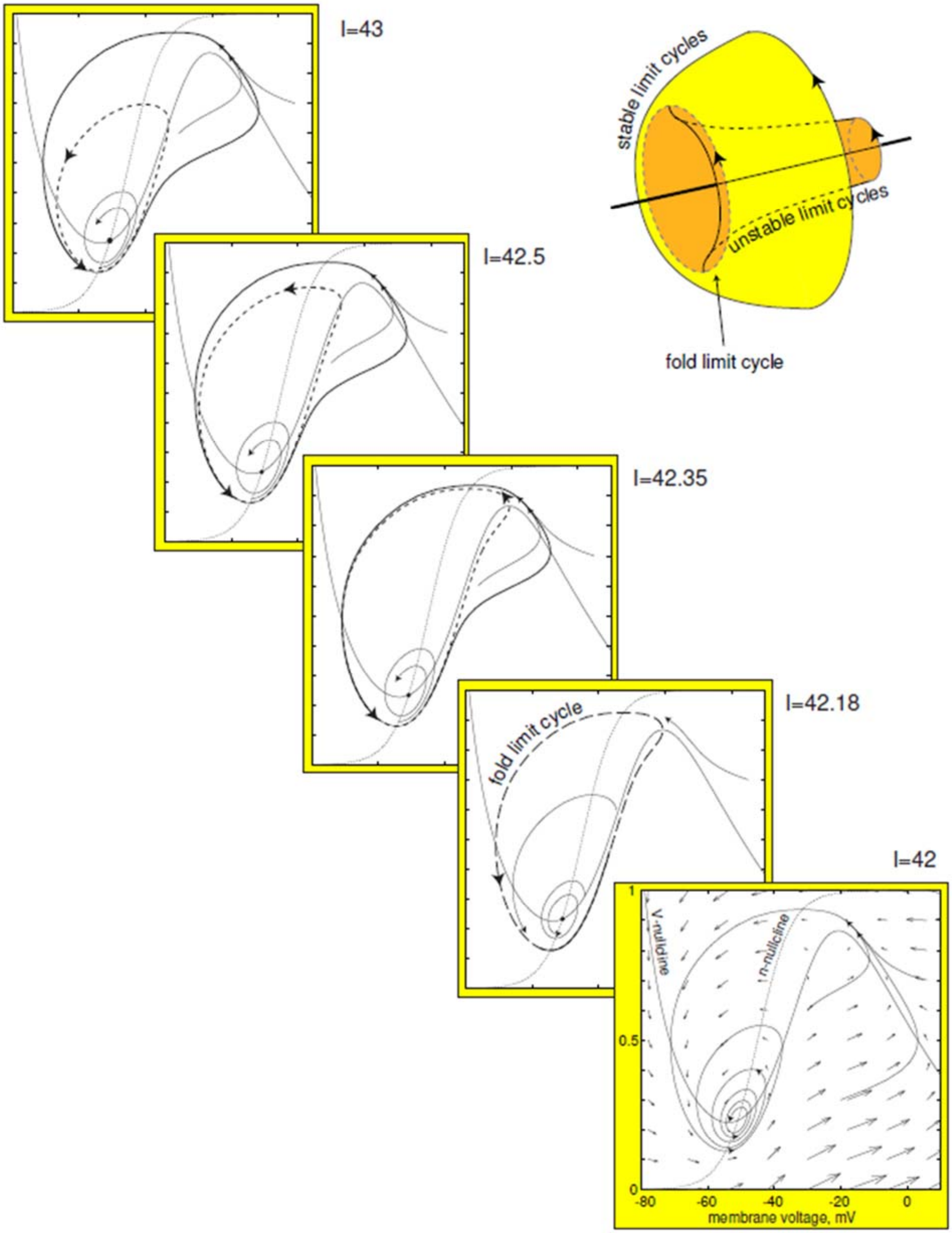


FIGURE 6.23. Fold limit cycle bifurcation in the $I_{\mathrm{Na,p}}+I_{\mathrm{K}}$-model. As the bifurcation parameter I decreases, the stable and unstable limit cycles approach and annihilate each other. Parameters as in Fig. 6.17.

Subcritical Andronov-Hopf Bifurcation:

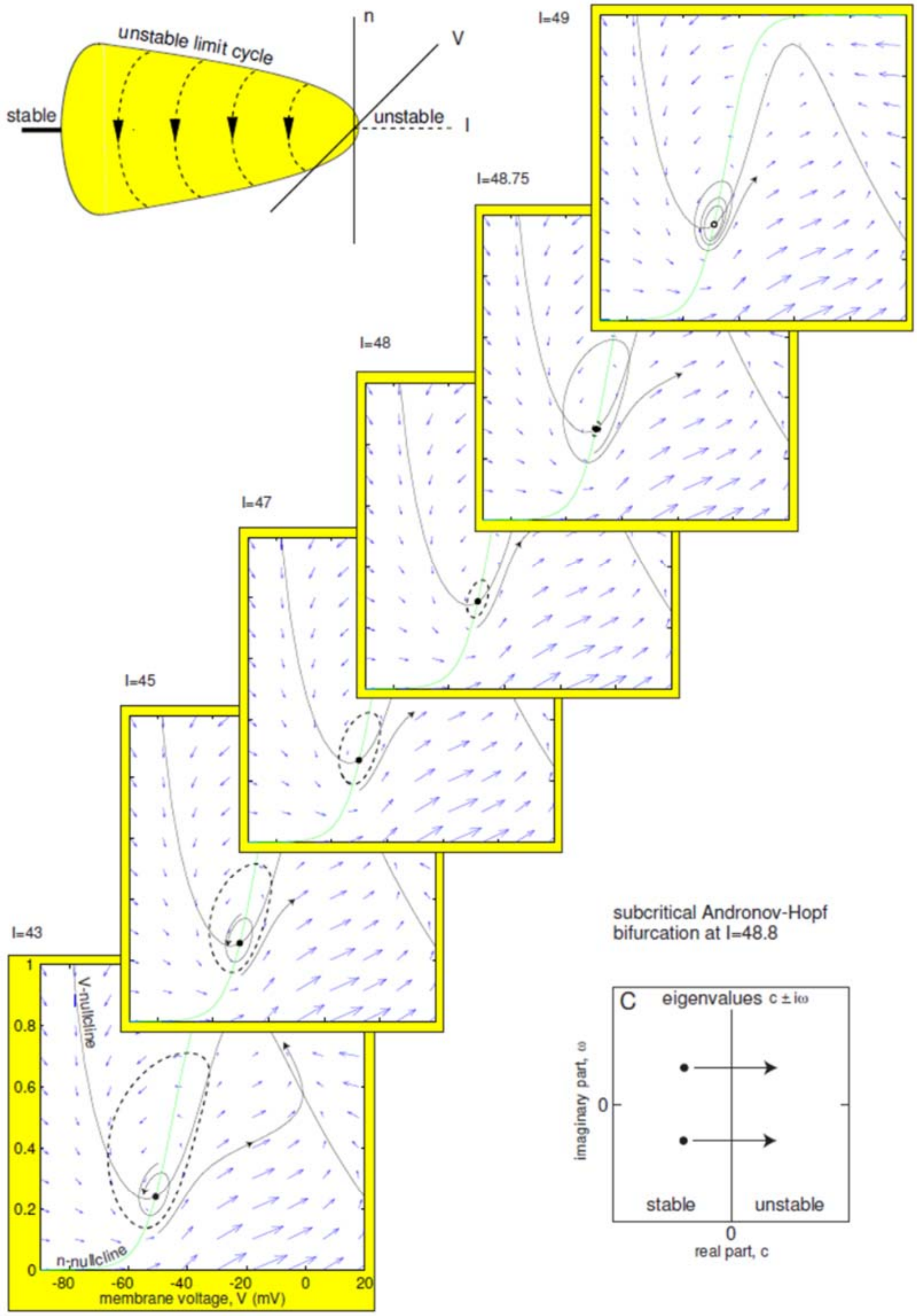


FIGURE 6.17. Subcritical Andronov-Hopf bifurcation in the $I_{\mathrm{Na,p}}+I_{\mathrm{K}}$-model: As a bifurcation parameter I increases, an unstable limit cycle (dashed circle; see also Fig. 6.16) shrinks to an equilibrium and make it lose stability. Parameters as in Fig. 4.1b except $g_{\mathrm{L}} = 1$, $g_{\mathrm{Na}} = g_{\mathrm{K}} = 4$, and Na^{+} activation function has $V_{1/2} = -30$ mV and $k = 7$.