Math 430 * Math 635 * Fall 2015 * Victor Matveev

Lecture 2: Physics Review

Quantity	Symbol	Explanation	Unit symbol	Units explanation
	Scalars			
Potential	V	$V = E_V / Q$	V (Volts)	V = J / Cb
Energy (electrostatic)	E_V	$\mathbf{F} = -\nabla E_V$	J (Joule)	J = N · m
Charge	Q	Fundamental	Cb (Coulomb)	Cb = A · s
Current	I	$I = dQ / dt$	A (Ampere)	A = Cb / s
Resistance	R	$R = V / I$	Ω (Ohm)	Ω = V / A
Conductance	G	$G = 1 / R = I / V$	S (Siemens)	S = 1/Ω = A / V
Conductivity	g	$G = G / Area$	S / m^2	Conductance per unit area
Capacitance	C	$C = Q / V$	F (Farad)	F = Cb / V
	Vectors			
Electric field strength	**E**	$\mathbf{E} = -\nabla V$	V / m = N / Cb	Check: (N · m)/Cb = J/Cb = V
Electrostatic force	**F**	$\mathbf{F} = -\nabla E_V$	N = kg m / s^2	N = J / m
Current density	**J**	ChargeDensity × **velocity**	A / m^2	Current per unit area

1. Potential = potential energy per charge (simple!): $\boxed{V = \frac{E_V}{Q}}$

   Therefore, 1 Joule of energy is released when a charge of one Coulomb crosses a potential difference of 1 Volt.

2. Potential energy is a property of conservative (potential) force, in this case, electric force:

$\mathbf{F} = -\nabla E \;\Rightarrow$ Force points in the direction of energy decrease

Divide both sides by Q: $\underbrace{\frac{\mathbf{F}}{Q}}_{\mathbf{E}} = -\nabla \underbrace{\left(\frac{E}{Q}\right)}_{V} \;\Rightarrow\; \boxed{\mathbf{E} = -\nabla V}$

$\Rightarrow$ Electric field (force per charge) points in the direction of potential decrease

1D case: $E_x = -\frac{\partial V}{\partial x}$

Example: field in the membrane pore of width ~ 4nm:

Assuming linear potential drop from 0 to -70mV, within the pore we obtain:

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\Delta V}{\Delta x} = \frac{70\,mV}{4\,nm} \approx 18 \cdot 10^6 \frac{V}{m} \quad \text{(huge!)}$$

3. Gauss law (part of Maxwell's equations) -- optional but good to know:

$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_o}$ Charge density creates a flux (divergence) of electric field

Plug in $\mathbf{E} = -\nabla V \;\Rightarrow\; \nabla^2 V = -\frac{\rho}{\varepsilon_o}$ (the Poisson equation)

4. Simplest circuit: R-circuit:

I = V / R = G V

Linear relationship between Current and Voltage (linear "I-V curve")