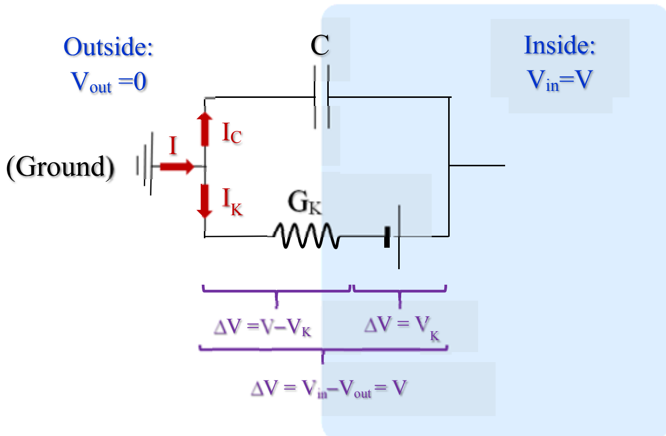
Math 430 * Math 635 * Fall 2015 * Victor Matveev

Equivalent circuit of a "passive" cell

Let's assume only K^+ channels are present:



Kirchhoff law:

total applied current (from outside to inside) = sum of currents through two branches of the circuit

$$I_C + I_K = I$$

$$\Rightarrow \quad I = C\frac{dV}{dt} + G_K\left(V - V_K\right) \quad \Rightarrow \quad \boxed{C\frac{dV}{dt} = I - G_K\left(V - V_K\right)}$$

Here $V_K = \frac{RT}{zF}\ln\frac{\left[K^+\right]_{out}}{\left[K^+\right]_{in}} = \frac{k_B T}{q}\ln\frac{\left[K^+\right]_{out}}{\left[K^+\right]_{in}}$ is the potassium reversal potential (Nernst potential)

• **NOTE: V_K<0, since a very negative potential is required to force K^+ ions to move into the cell against their concentration gradient.** See proof of this expression in another file…

We can easily generalize to two different channel types are present, K^+ and Cl^-:

$$I_C + I_K + I_{Cl} = I$$

$$\Rightarrow \quad I = C\frac{dV}{dt} + G_K\left(V - V_K\right) + G_{Cl}\left(V - V_{Cl}\right) \quad \Rightarrow \quad \boxed{C\frac{dV}{dt} = I - G_K\left(V - V_K\right) - G_{Cl}\left(V - V_{Cl}\right)}$$

Let's who that the two (or any number) of ionic currents can be combined into a single term:

$$C\frac{dV}{dt} = I - G_K(V - V_K) - G_{Cl}(V - V_{Cl})$$

$$= I - (G_K + G_{Cl})V + G_K V_K + G_{Cl} V_{Cl}$$

$$= I - (G_K + G_{Cl})\left(V - \frac{G_K V_K + G_{Cl} V_{Cl}}{G_K + G_{Cl}}\right)$$

$$= I - G_L(V - V_R)$$

Where we introduce two quantities:

- Total "leak" / "input" conductance: $\boxed{G_L = G_K + G_{cl}}$
- "Resting" / "equilibrium" potential: $\boxed{V_R = \frac{G_K V_K + G_{Cl} V_{Cl}}{G_K + G_{Cl}}}$

  = average of reversal potentials weighted by their conductances

We may perform another simplification:

$$C\frac{dV}{dt} = I - G_L(V - V_R)$$

$$= -G_L\left(V - V_R - \frac{I}{G_L}\right)$$ Now let's denote $\frac{1}{G_L} = R_L$ ("Input resistance")

$$= -G_L\left(V - (V_R + R_L I)\right)$$

$$= -G_L(V - V_{RI})$$ where equilibium potential in non-zero current is $V_{RI} = V_R + R_L I$

Finally, we can divide both sides of the equation by C, and note that $G_L / C = 1 / (R_L C) = 1 / \tau_m$

$$\frac{dV}{dt} = -\frac{V - V_{RI}}{\tau_m}$$

Denoting the deviation from equilibrium as $v = V - V_{RI}$, and noting that $dv = dV$, we obtain:

$$\frac{dv}{dt} = -\frac{v}{\tau_m} \Rightarrow \boxed{v(t) = v_o e^{-t/\tau_m}} \Rightarrow V(t) - V_{RI} = (V_o - V_{RI})e^{-t/\tau_m}$$

$$\Rightarrow \boxed{V(t) = V_{RI} + (V_o - V_{RI})e^{-t/\tau_m}}$$ potential decays exponentially to V_{RI} with characteristic time τ_m