Fall 2015 * Math 430 / 635 * Midterm Examination * October 26, 2015 * Victor Matveev

This is a closed-book exam; neither notes nor calculators allowed. Please explain all steps in your work.
<u>You can drop one problem among 2-5</u>

1. **(20pts)** Write down the sequence of X values that this Matlab script prints out during execution:

```
X = 3;
for K = 1 : 5
   X = 2 * X / K;
   if  K > 4
      X = 1 + X * 5;
   end
   disp(X);    %   This command displays the value of X
end
```

2. **(20pts)** Suppose that the cell with $[Ca^{2+}]_{in}$=0.1 μM is immersed in a bath with $[Ca^{2+}]_{out}$=10 μM, and that the corresponding Nernst potential for Ca^{2+} equals +50 mV. The concentration of Ca^{2+} in the bath (i.e. $[Ca^{2+}]_{out}$) is then increased, raising the Nernst potential to +100 mV. Assuming that the temperature of the bath was not changed, what is the new value of $[\mathrm{Ca}^{2+}]_{\mathrm{out}}^{\mathrm{new}}$ corresponding to this higher value of the Nernst potential?

3. **(20pts)** Write down the passive membrane equation and use it to obtain the expression for the inter-spike interval of a linear integrate-and-fire model driven by a constant current I_0. Finally, substitute the following parameter values into your expression, and simplify as much as possible, expressing your answer in terms of a logarithm of a dimensionless number: $I_o = 20\,\mathrm{nA},\ \tau_m = 10\ \mathrm{ms},\ R = 5\ \mathrm{M}\Omega,\ V_R = -60\,\mathrm{mV},\ V_{reset} = -70\,\mathrm{mV},\ V_{th} = -50\,\mathrm{mV}$

4. **(20pts)** Consider the following non-linear differential equation: $\begin{cases} Y' = \dfrac{1}{Y} - 1 \\ Y(0) = 2 \end{cases}$

   a) Find the equilibrium, and categorize its stability
   b) Make a rough sketch of the solution, $Y(t)$
   c) Use Euler method to find Y_1 and Y_2, using h=1. Place the corresponding points on the solution sketch that you made in part "b"

5. **(20pts)** Consider the following parameter-dependent system: $V' = \dfrac{4}{1+V^2} - I$

   a) Find all equilibria as a function of parameter I; note whether they exist for all values of I or not.
   b) Find critical (bifurcation) value I_{cr}, and examine the stability of all equilibria.
   c) Sketch the bifurcation diagram, V^* vs I, indicating stable equilibria with a solid curve, and unstable equilibria with a dashed curve. You don't have to examine the non-degeneracy and transversality conditions.

6. **(20pts)** Consider the following system: $\begin{cases} x' = 4\ln y \\ y' = e^x - 1 \end{cases}$

   a) Find the equilibrium, calculate the Jacobian at this equilibrium, and categorize its stability
   b) Sketch the nullclines (they are very simple!). Then, sketch the flow field along the nullclines, and in the entire x-y phase-plane
   c) Find the eigenvectors of the Jacobian at the equilibrium, and use your plot in part "b" to illustrate their meaning