Math 613 * Fall 2018 * Victor Matveev * Homework 3

1. Categorize the stability of the equilibrium for the following systems by examining the eigenvalues of the linearization (you can use the trace-determinant diagram if you like). In (c), you have to start by finding the equilibrium and the Jacobian at equilibrium:

$$(a)\ \begin{cases} \dfrac{dx}{dt} = -2x + y \\ \dfrac{dy}{dt} = x - 4y \end{cases} \qquad (b)\ \begin{cases} \dfrac{dx}{dt} = -2x + y \\ \dfrac{dy}{dt} = -x + 3y \end{cases} \qquad (c)\ \begin{cases} \dfrac{dx}{dt} = \exp(x - 5y) - 1 \\ \dfrac{dy}{dt} = \sqrt{1 + 2x - 6y} - 1 \end{cases}$$

2. Consider the following linear system (this system corresponds to the $\text{Trace}^2(J) = \text{Det}(J)/4$ curve):

$$\begin{cases} \dfrac{dx}{dt} = -x + y \\ \dfrac{dy}{dt} = -y \\ x(0) = 0;\ y(0) = 1 \end{cases}$$

   a) Perform linear stability analysis. Is the equilibrium stable? Is it asymptotically stable?
   b) Sketch the full vector field / direction field on the rectangular domain (-1, 1) x (-1, 1); indicate any nullclines, and any eigenvectors/eigendirections
   c) Use your vector field sketch to plot the trajectory in phase space (x vs. y) for the given initial condition, and then plot x(t) and y(t) as functions of time.

3. Consider the following ODE in R^2:

$$\begin{cases} \dfrac{dx}{dt} = y^2 \\ \dfrac{dy}{dt} = x^2 \end{cases}$$

   Since both eigenvalues of the Jacobian at equilibrium are zero, linear stability analysis does not give any information about its behavior. Sketch the vector field to determine the stability of equilibrium (hint: start with sketching the flow along the nullclines and the diagonals $x = y$ and $x = -y$)