Math 613 * Fall 2018 * Victor Matveev * Homework 7

Boldface quantities are constant vectors or vector fields; *italic* quantities are scalars

1. Combine the divergence theorem with the Gauss law $\nabla \cdot \mathbf{E}(\mathbf{r}) = \dfrac{\rho(\mathbf{r})}{\varepsilon_0}$ to find the electric field strength $\mathbf{E}(\mathbf{r})$ *inside* a uniformly charged ball of radius r_0 with total charge Q.

Hints:

a) Start by calculating the density ρ for a total charge Q uniformly distributed over a ball of radius r_o

b) Assume that the electric field is oriented along radial lines, and that its magnitude depends only on the distance from the origin: $\mathbf{E}(\mathbf{r}) = E(r)\hat{\mathbf{r}},$ where $E = |\mathbf{E}|,\ r = |\mathbf{r}|,\ \hat{\mathbf{r}} = \dfrac{\mathbf{r}}{|\mathbf{r}|}$

c) Apply the Divergence Theorem to the Gauss law, integrating over a ball with a radius *smaller* than the radius r_o

2. Combine the divergence theorem with the Gauss law $\nabla \cdot \mathbf{E}(\mathbf{r}) = \dfrac{\rho(\mathbf{r})}{\varepsilon_0}$ to find the electric field strength $\mathbf{E}(\mathbf{r})$ *outside* an infinitely long cable of radius r_0 with uniform charge density ρ

Hints:

a) Assume that the electric field is oriented away from the cable axis lines, and that its magnitude depends only on the distance from this axis: $\mathbf{E}(\mathbf{r}) = E(r)\hat{\mathbf{r}},$ where $E = |\mathbf{E}|,$ $\hat{\mathbf{r}}$ is a unit vector pointing away from the axis of the cable.

d) Apply the Divergence Theorem to **E**(**r**), using a *cylindrical* surface with a radius *larger* than the radius r_o

Next problem will be carried over to homework #8:

3. Re-write the following expressions using suffix notation (do not simplify):

$a)$ $\operatorname{trace}(AB)$ $\quad b)$ $\nabla \cdot (\mathbf{u}(\mathbf{r}) \times \mathbf{v}(\mathbf{r}))$ $\quad c)$ $\det(A)$ (hint: see your answer to part "b")