Math 613
Homework #10

1. (10pts) Convert to index notation: $\boldsymbol{a} \cdot \boldsymbol{b} \times \boldsymbol{c} + (\boldsymbol{a} \cdot \boldsymbol{c})\,(\boldsymbol{a} \cdot \boldsymbol{b}) = 3\,|\boldsymbol{c}|^2$

2. (10pts) Simplify and write down the final result in vector notation:

$$u_j\, b_k\, u_m\, \delta_{m\,k} + b_k\, c_m\, u_n\, c_k\, \varepsilon_{j\,m\,n} = \delta_{k\,l}\, a_m\, \delta_{l\,j}\, a_k\, b_m$$

3. (20pts) Simplify the following expressions:

a) $\delta_{k\,n}\,\delta_{j\,k}\,\delta_{n\,j}$ (be careful with the final simplification step)

b) $\varepsilon_{j\,k\,m}\,\delta_{k\,n}\,\delta_{m\,j}$

4. (15pts) Re-write using suffix notation, use the standard product rule, and convert the result back to vector notation: $\nabla \cdot (\mathbf{U} \times \mathbf{V})$

5. (15pts) Use suffix notation to prove that $\nabla \times (\nabla \times \mathbf{U}) = \nabla(\nabla \cdot \mathbf{U}) - \nabla^2 \mathbf{U}$

6. (30pts) Calculate the "convective acceleration" $\mathbf{A} = (\mathbf{U} \cdot \nabla)\mathbf{U}$ for the "2D" ($\mathbb{R}^2$) flow $\mathbf{U} = (y^2,\ x^2)$ (you don't have to use suffix notation). Make separate plots of **U** and **A**

Below is everything you need to know about suffix / index / Einstein notation (here *A* denotes a tensor of *any* rank, i.e. a vector, a matrix, rank-3 tensor, etc):

$$\delta_{jk}\ A_{k\,\ldots\ldots} = A_{j\,\ldots\ldots}$$

$$\nabla \equiv \frac{\partial}{\partial x_k} \equiv \partial_k$$

$$(\mathbf{U} \times \mathbf{V})_k = \varepsilon_{k\,m\,n}\, U_m V_n$$

$$\varepsilon_{k\,j\,m}\ A_{j\,m\ldots\ldots} = 0 \ \text{ if } \ A_{j\,m\ldots\ldots} = A_{m\,j\ldots\ldots}$$

$$\varepsilon_{k\,j\,m} = \varepsilon_{j\,m\,k} = \varepsilon_{m\,k\,j} = -\varepsilon_{j\,k\,m} = -\varepsilon_{k\,m\,j} = -\varepsilon_{m\,j\,k}$$

$$\varepsilon_{k\,j\,i}\ \varepsilon_{k\,m\,n} = \delta_{j\,m}\,\delta_{i\,n} - \delta_{j\,n}\,\delta_{i\,m}$$