**Math 613 * Fall 2019 * Victor Matveev * Homework 3**

**1.** The non-dimensionalized equation of the damped pendulum in the case of small angle reads $\frac{d^2\theta}{dt^2}+\gamma\frac{d\theta}{dt}+\theta=0$

a) Show that this 2nd-order ODE can also be written as a system of 1st-order ODEs: $\begin{cases}\frac{dx}{dt}=y\\ \frac{dy}{dt}=-x-\gamma y\end{cases}$

b) Use linear stability analysis to show that this system can have somewhat different behavior, depending on the value of the damping parameter $\gamma$. You can assume that this damping parameter is always positive: $\gamma > 0$

c) Explain in **one** sentence each of the behaviors you found in part "b", in terms of the pendulum motion.

**2.** Categorize the stability of all equilibria for the following non-linear systems by examining the eigenvalues of the linearization (the Jacobian) near each equilibrium. You can use the trace-determinant diagram if you like, or just calculate the eigenvalues of the Jacobian directly. Start by finding all equilibria and the Jacobian at each equilibrium. In part (b), it is easy to find the Jacobian using the Taylor series for ln(1 + x) and 1/(1 – x) (see previous homework)

$$(a)\begin{cases}\frac{dx}{dt}=1-y\\ \frac{dy}{dt}=x^2-y\end{cases}\qquad (b)\begin{cases}\frac{dx}{dt}=\ln(1+x+5y)\\ \frac{dy}{dt}=\frac{1}{1-x-2y}-1\end{cases}$$

**3.** Sketch the flow field of the system in problem 2(a). Start by sketching the flow along the nullclines and the coordinate axes.

**4.** Consider the following ODE in $R^2$: $\begin{cases}\frac{dx}{dt}=y-xy-x^2\sin x\\ \frac{dy}{dt}=x^2-x-\sin^3 y\end{cases}$

Show that the linear stability analysis gives no information about the stability of the equilibrium at (0, 0). However, you will easily determine the stability of this zero equilibrium by examining the time change of the distance from the origin, using the observation that $\frac{d}{dt}r^2=\frac{d}{dt}\left(x^2+y^2\right)=2\left(x\frac{dx}{dt}+y\frac{dy}{dt}\right)$. Keep only the lowest-order non-zero term in the Taylor expansion of the right-hand side of this equation for $d(r^2)/dt$. This is a simple example of the so-called Lyapunov function method (I will show another example on Monday).