

Math 111 EXAM II, April 9, 2003

Read each problem carefully. Show all your work for each problem! No Calculators!

1. (20) Find  $d^2y/dx^2$  for the following:

$$(a) y = x^2 + \frac{1}{x^2}, \quad (b) y = \tan(x), \quad (c) y = \cos(\sin(x)), \quad (d) y = 2 \sin(y - x) + x.$$

2. (a) (10) Find two positive real numbers,  $x$  and  $y$ , such that their product is 25 and their sum is as small as possible. Verify this value is a minimum.

- (b) (10) Find an approximation to the root of the function  $f(x) = x^3 + 3x + 1$  using Newton's Method. Use  $x_0 = 0$  and calculate the first iteration,  $x_1$ .

3. (a) (10) Find the linear approximation of the function  $f(x) = x^2$  near the point  $a = 0$ , and then near the point  $a = -1$ .

- (b) (5) Show that the function  $f(x) = x^3$  satisfies the hypotheses of the mean value theorem on the interval  $x \in [-1, 1]$ . Find all numbers,  $c$ , in this interval that satisfy the conclusion of that theorem.

4. (a) (5) Find all horizontal, vertical and slant asymptotes for the function

$$y = \frac{x^2}{x - 1}.$$

- (b) (10) Each side of an equilateral triangle is decreasing at a rate of 2 in/s. At what rate is the area decreasing when the area is  $\sqrt{3}$  in<sup>2</sup>.

5. (15) For the function below, find all of the following if they exist: (i) all local extrema, (ii) points of inflection, (iii) intervals where the function is increasing or decreasing, (iv) intervals of upward and downward concavity, and (v) all asymptotes. Also, sketch a plot of the curve  $y = f(x)$ .

$$f(x) = 1 + \frac{1}{x} - \frac{1}{x^2}.$$

6. (a) (5) Evaluate the indefinite integral

$$\int x(x + 1) dx.$$

- (b) (10) The acceleration,  $a(t)$ , and initial velocity,  $v(0)$ , of a particle for the time interval  $t \in [0, 4]$  are given by

$$a(t) = \frac{1}{\sqrt{t}} - \sqrt{t}; \quad v(0) = 2.$$

What are the particle's maximum and minimum velocities on this interval?